Solution of Multi-Objective Interval Solid Transportation Problems using Expected Value

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Abstract: *In this paper, a solution procedure has been given for the Multi-Objective Interval Solid Transportation Problem under stochastic environment where the cost coefficients of the objective functions, source availability, destination demand and conveyance capacities have been taken as stochastic intervals. The problem has been transformed into a classical multi-objective transportation problem where the multiple objective functions are minimized by using fuzzy programming approach. Numerical examples are provided to illustrate the approach.*

Keywords: *Solid transportation problem; Multi-objective interval solid transportation problem; stochastic programming; Fuzzy programming*

1. Introduction

As a generalization of traditional TP, the Solid Transportation Problem (STP) was stated by Shell [1] in 1955, which, he considered the three item properties in the constraint set instead of two items namely source and destination. He also suggested the situations where the STP would arise, and four cases of STP were discussed according to the data given on the item properties and developed its solution procedure. Basu et al. [2] developed an algorithm for finding the optimum solution for the solid fixed charge linear transportation problem. Although STP was forgotten for long time, because of existing advanced solution methodologies, recently it is receiving the attention of many researchers of this field. Models and algorithms have been developed by many authors [3-9].

In literature, it was found that various effective algorithms were developed for solving transportation problems with the assumption that the coefficients of the objective function, source availability, destination demand and conveyance capacities are specified in a crisp manner. However, these conditions may not be satisfied always. Since in the present situation, the unit transportation costs are rarely constant. To deal the problems with ambiguous coefficients in mathematical programming, inexact and interval programming techniques have been developed by many authors [10- 13].

The STP in uncertain environment becomes important branch of optimization and a lot of models and algorithms have been presented for different problems by different authors [14, 15, 16, 17]. A.Nagarajan and K.Jeyaraman developed many models and methods for solving multi_ objective interval solid Transportation problems in stochastic environment[20,26,27,28,29,30],solution procedures for solid fixed cost bi-criterion indefinite quadratic transportation problem under stochastic environment [20]. S.K.Das et al. [21], developed the theory and methodology for multi-objective transportation problem with interval cost, source and destination parameters. Expected value of fuzzy variable and fuzzy expected value models presented by Baoding Liu and Yian-Kui Liu [22].

The fuzzy set theory concept was first introduced by Zadeh [23]. Linear programming problems with several objective functions was solved by using fuzzy membership functions by Zimmerman [24] and he showed that the results obtained from fuzzy are always efficient. A special type of non-linear membership function was used for the vector maximum linear programming problem [25].

In this paper, the idea of stochastic environment has been employed for MOISTP a method has been proposed to solve the MOISTP. Using expectation of random variables, we have constructed an equivalent crisp model to the given MOISTP. To obtain the solution of this equivalent problem, we have used fuzzy programming approach. In order to illustrate the proposed method, numerical examples are provided.

This paper is organized as follows. In Section 2, the basic idea of MOISTP has been given. In Section 3, definitions of interval arithmetic and related definitions have been given. The formulation of crisp objective function and crisp constraint have been given in the section-4 and section-5 respectively. Expected value of MOISTP is given in Section-6. Fuzzy programming approach for the solution of MOISTP is given in Section-7. The numerical example is given in section 8 along with the solution to illustrate the approach.

2.Multi-objective Interval Solid TransportationProblem (MOISTP)

The MOISTP is a generalization of the multi-objective solid transportation problem in which input data are expressed as stochastic variables as well as stochastic intervals instead of point values. These types of problems arise only when uncertainty occurs in data. The decision makers consider it as more convenient to express it as intervals which can be stated as follows.

Problem-I :
Minimize

$$Z^p = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} [c^p_{Lijk}, c^p_{Rijk}]\, x_{ijk},$$

$$p = 1, 2, 3,..., P \quad (1)$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = [a_{Li}, a_{Ri}],$$

$$i = 1, 2, 3,\ldots, m. \quad (2)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = [b_{Lj}, b_{Rj}],$$

$$j = 1, 2, 3, ..., n. \quad (3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = [e_{Lk}, e_{Rk}],$$

$$k = 1, 2, 3, ... ,l. \quad (4)$$

with $\sum_{i=1}^{m} a_{Li} \geq \sum_{j=1}^{n} b_{Lj}$, $\sum_{i=1}^{m} a_{Ri} \geq \sum_{j=1}^{n} b_{Rj}$, $\sum_{k=1}^{l} e_{Lk} \geq \sum_{j=1}^{n} b_{Lj}$, $\sum_{k=1}^{l} e_{Rk} \geq \sum_{j=1}^{n} b_{Rj}$ (non-balanced condition). (5)

Where $[c^p_{Lijk}, c^p_{Rijk}]$ for p = 1, 2, 3,..., P are intervals representing the uncertain cost for the transportation problem; it can represent delivery time, quantity of goods delivered, under used capacity, etc. The source parameter lies between left limit a_{Li} and right limit a_{Ri}, similarly, destination parameter lies between left limit b_{Lj} and right limit b_{Rj} and conveyance parameter lies between left limit e_{Lk} and right limit e_{Rk}.

Definition 2.1 [18, 19]

Let $* \in (., /, +, -)$ be a binary operation on the set of real numbers. If A and B are closed intervals, then

$$A * B = \{ a*b: a \in A, b \in B \} \quad (6)$$

defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. The interval operations used in this research paper are as given below.

$$A + B = [a_L, a_R] + [b_L, b_R]$$
$$= [a_L + b_L, a_R + b_R], \quad (7)$$

$$A + B = \langle a_C, a_W \rangle + \langle b_C, b_W \rangle$$
$$= \langle a_C + b_C, \quad a_W + b_W \rangle, \quad (8)$$

$$kA = k[a_L, a_R]$$
$$= [ka_L, ka_R] \text{ for } k \geq 0, \quad (9)$$

$$kA = k[a_L, a_R]$$
$$= k[a_R, \quad ka_L] \quad \text{for} \quad k < 0, \quad (10)$$

$$KA = k\langle a_C, a_W \rangle = \langle ka_C, \quad |k| a_W \rangle, \quad (11)$$

where 'k' is real number.

3. Order relation between Intervals

The order relations which represent the decision makers' preference between interval costs are defined for the minimization problems. Let the uncertain costs from two alternatives be represented by intervals 'A' and 'B' respectively. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

Definition 3.1 The order relation $\leq_{LR}$ between A = $[a_L, a_R]$ and B =$[b_L, b_R]$ is defined as

$A \leq_{LR} B$ iff $a_L \leq b_L$ and $a_R \leq b_R$, $A <_{LR} B$ iff $A \leq_{LR} B$ and $A \neq B$.(12)

This order relation $\leq_{LR}$ represents the decision makers' preference for the alternative with lower minimum cost and maximum cost, i.e., if $A \leq_{LR} B$, then A is preferred to B.

Definition 3.2 The order relation $\leq_{CW}$ between A = $\langle a_C, a_W \rangle$ and

B = $\langle b_C, b_W \rangle$ is defined as

$A \leq_{CW} B$ iff $a_C \leq b_C$ and $a_W \leq b_W$ $A <_{CW} B$ iff $A \leq_{CW} B$ and $A \neq B$.(13)

This order relation $\leq_{CW}$ represents the decision makers' preference for the alternative with lower expected cost and less uncertainty, i.e., if A $\leq_{CW}$ B, then A is preferred to B.

4. Formulation of the crisp objective function

In this section, the formulation of original interval objective function has been made as a crisp one.

Definition 4.1 $x_{ijk}^{\ 0} \in$ S is an optimal solution of the problem-I iff there is no other solution $x_{ijk} \in$ S which satisfies Z(x) $<_{LR}$ Z(x^0)or Z(x) $<_{CW}$ Z(x^0).

Theorem 4.1 It can be proved that A $\leq_{RC}$ B iff A $\leq_{LR}$ B or A $\leq_{CW}$ B,A $<_{RC}$ B iff A $<_{LR}$ B or A $<_{CW}$ B, (14)

where the order relation $\leq_{RC}$ is defined as

A $\leq_{RC}$ B iff $a_R \leq b_R$ and $a_C \leq b_C$, A $<_{RC}$ B iff A $\leq_{RC}$ B and A $\neq$ B.

Using the theorem 4.1, Definition 4.1 is simplified as follows.

Definition 4.2 $x^0 \in$ S is an optimal solution of the Problem-I iff there is no other solution x $\in$ S which satisfies

Z(x) $<_{RC}$ Z(x^0).The right limit

Z_R^p(x) of the interval objective function in problem-I is derived from the equations (8) and (11) as

$$Z_R^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^p x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^p \left|x_{ijk}\right| \qquad (15)$$

where c_{Cijk}^p is the centre and c_{Wijk}^p is the half width of the coefficient of x_{ijk} in Z^p.

In the case when $x_{ijk} \geq 0$, i = 1, 2, 3,…, m, j = 1, 2, 3, ..., n, k = 1, 2, 3, …, l,

Z_R^p(x) is modified as:

$$Z_R^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^p x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^p x_{ijk}. \qquad (16)$$

The centre of the objective function Z_C^p(x) for the Problem–I can be defined as

$$Z_C^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^p x_{ijk}. \qquad (17)$$

The solution set of the Problem-I defined by Definition 4.2 is also obtained as the Pareto optimal solution of the two multi-objective problem as:

Minimize { Z_R^p, Z_C^p }, p = 1, 2, 3,…,P,

subject to the constraints (2) – (5) respectively where Z_R^p and Z_C^p are as stated as in equations (16) and (17).

5. Formulation of the crisp constraint

By using the theory of interval arithmetic [18,19], the Problem-I is converted into its equivalent form as follows .

Problem -II:

Minimize

$$Z^p = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} [c_{Lijk}^p, c_{Rijk}^p]\, x_{ijk}, \quad p= 1, 2, 3,..., P \qquad (18)$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq a_{Li}, \quad i = 1, 2, 3,…,m. \qquad (19)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq a_{Ri}, \quad i = 1, 2, 3,…,m. \qquad (20)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq b_{Lj}, \quad j = 1, 2, 3, ..., n. \qquad (21)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq b_{Rj}, \quad j = 1, 2, 3, ..., n. \qquad (22)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq e_{Lk}, \quad k = 1,2,3,…, l. \qquad (23)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq e_{Rk}, \quad k=1,2,3,…,l. \qquad (24)$$

$x_{ijk} \geq 0$,for all i, j, k. $\sum_{i=1}^{m} a_{Li} \geq \sum_{j=1}^{n} b_{Lj}$ $\sum_{i=1}^{m} a_{Ri} \geq \sum_{j=1}^{n} b_{Rj}$, x_{ijk}^{*} $e_{Lk} \geq \sum_{j=1}^{n} b_{Lj}$, $\sum_{k=1}^{l} e_{Rk} \geq \sum_{j=1}^{n} b_{Rj}$

(non balanced condition). (25)

6. Expected value of stochastic variable

In this section, the expected value of a stochastic variable is defined.

Definition 6.1 Let 'ξ' be a random variable whose expected value is defined by

$$E[\xi] = \int_{0}^{+\infty} \Pr\{\xi \geq r\}dr - \int_{-\infty}^{0} \Pr\{\xi \leq r\}dr$$

provided that at least one of the two integrals is finite where r $\in \Re$ [14].

Let 'ξ' and 'η' be random variables with finite expected values. For any values of 'a' and' b' it has been proved that

$E[a\xi + b\eta] = aE[\xi] + bE[\eta]$. i.e, the expected value operator has the linearity property.

Theorem 6.1 Let 'ξ' be a random variable whose probability density function 'ϕ' exists. If the Lebesgue integral $\int_{-\infty}^{+\infty} x\phi(x)dx$ is finite, then it is arrived as $E[\xi] = \int_{-\infty}^{+\infty} x\phi(x)dx$.

6.1 Expected value model for MOISTP

The expected value model (EVM) which optimizes some expected objective function subjected to some expected constraints, for example ,minimizing the expected time, minimizing expected cost, maximizing expected profit etc. Normally if we want to find a decision with maximum expected return subjected to some expected constraints then we have the following EVM,

max $E[f(x_{ijk}, \xi)]$ subject to

$E[g_j(x_{ijk}, \xi)] \leq 0$, j = 1,2,3,…, p

Where . x_{ijk} is a desicion vector , ξ is a stochastic vector, $f(x_{ijk}, \xi)$ is the return function, $g_j(x_{ijk}, \xi)$ are stochastic constrained functions for j = 1,2,3,…, q.

Definition 6.2

A solution x_{ijk} is feasible if and only if $g_l(x_{ijk}, \xi) \leq 0$, j = 1,2,3,…, p. A fesiable solution is an optimal solution to EVM if $E[f(x^*_{ijk}, \xi)] \geq E[f(x_{ijk}, \xi)]$ for any feasible solution $x_{ijk.}$

In n multiple objective problems we employ the following expected value multiobjective programming(EVMOP).

max $E[f_1(x_{ijk}, \xi),]$, $E[f_2(x_{ijk}, \xi),]$
$E[f_3(x_{ijk}, \xi),]$ ….$E[f_t(x_{ijk}, \xi),]$ subject to
$E[g_l(x_{ijk}, \xi)] \leq 0$, l = 1,2,3,…, q where
$f_r(x, \xi)$ are return functions for
r = 1,2,3,….t.

Definition 6.3

A feasible solution x^*_{ijk} is said to be pareto optimal solution to EVOMP if there is no feasible solution x_{ijk} such that

$E[f_i(x_{ijk}, \xi),] \geq E[f_1(x^*_{ijk}, \xi),]$
r = 1,2,3,….t

and $E[f_l(x_{ijk}, \xi),] > E[f_i(x^*_{ijk}, \xi),]$ for at least one index l.The expected value model for the problem II defined in section 5 takes the form

Problem - III: Minimize

$$Z^p = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} [c^p_{Lijk}, c^p_{Rijk}]\, x_{ijk}$$

p = 1, 2, 3,…,P,

subject to:

$E[\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} - a_{Li}] \geq 0$, i =1,2,3,…, m.

$E[\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} - a_{Ri}] \leq 0$, i =1,2,3,…, m.

$E[\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} - b_{Lj}] \geq 0$, j =1,2,3, ..., n. $E[\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} - b_{Rj}] \leq 0$, j =1,2,3,..., n. $E[\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} - e_{Lk}] \geq 0$, k=1,2,3,…, l. $E[\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} - e_{Rk}] \leq 0$, k =1,2,3, …, l.

where $x_{ijk} \geq 0$, for any i, j, k.

The problems proposed in the previous sections are constructed under stochastic environment. In order to find the suitable solution for the problems, the expected value, critical value or credibility measure must be calculated. If the stochastic parameters are complex, the computing objective values subject to the constraints becomes a time consuming one. Due to this, it is better to convert the models into their crisp equivalents by using the appropriate probability levels defined by the decision makers.

By using the linearity of expected value operator of random variable, the Problem–III is equivalent to
:

Problem IV: Minimize

$$Z^p = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}[c^p_{Lijk}, c^p_{Rijk}]x_{ijk}$$

p = 1, 2, 3,…,P,

subject to:

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \geq E[a_{Li}],\ i = 1,23,\ldots, m.$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \leq E[a_{Ri}],\ i = 1,2,3,\ldots, m.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \geq E[b_{Lj}],\ j = 1,2,3,\ldots, n.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \leq E[b_{Rj}]\ j = 1,2,3,\ldots, n.$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \geq E[e_{Lk}],\ k = 1,2,3,\ldots, l.$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \leq E[e_{Rk}],\ k = 1,2,3,\ldots, l.$$

where $x_{ijk} \geq 0$, for any i, j, k.

7.FuzzyProgramming approach for MOISTP

The MOISTP can be considered as a vector minimum problem. The first step to solve the problem is to assign, for each objective, two values U^p and L^p as upper and lower bounds, respectively, for the p-th objective, where U^p is the highest acceptable level for achievement for the p-th objective, L^p is the aspired level of achievement for the p-th objective and $d^p = U^p - L^p$ is the degradation allowance for the p-th objective. Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model and then convert the fuzzy model into a crisp model. The steps of the fuzzy programming approach may be summarized as follows.

Algorithm:

Step 1. Solve the multi-objective interval solid transportation problem using one objective at a time(ignoring all others) subject to the given set of constraints by using any one of the suitable evolutionary technique. Let $X^{1*} = \{x^1_{ijk}\}$, $X^{2*} = \{x^2_{ijk}\}$, $X^{3*} = \{x^3_{ijk}\}$,…, $X^{P*} = \{x^p_{ijk}\}$ be the optimum solutions for P different single objective interval solid transportation problems.

Step 2. From the results of step1, the values of all the objective functions will be calculated at all these 'P' optimal points. Then a payoff matrix is formed. The diagonal of the matrix constitutes individual optimum minimum values for the P objectives.The 'X^{P*}''s are the individual optimal solutions and each of these are used to determine the values of other individual objectives, thus the payoff matrix is developed as follows:

$$\begin{array}{c} \\ Z^1 \\ Z^2 \\ Z^3 \\ \\ Z^p \end{array}\begin{array}{c} \begin{array}{cccc} X^{1*} & X^{2*} & \ldots & X^{P*} \end{array} \\ \begin{pmatrix} Z^1(X^{1*}) & Z^1(X^{2*}) & \ldots & Z^1(X^{P*}) \\ Z^2(X^{1*}) & Z^2(X^{2*}) & \ldots & Z^2(X^{P*}) \\ Z^3(X^{1*}) & Z^3(X^{2*}) & \ldots & Z^3(X^{P*}) \\ & & & \\ Z^p(X^{1*}) & Z^p(X^{2*}) & \ldots & Z^p(X^{P*}) \end{pmatrix}\end{array}$$

We find the upper and lower bound for each objective from the payoff matrix. Here $L^p = Z^p(X^{P*})$ and $U^p = \max\{$
$Z^p(X^{1*}), Z^p(X^{2*}), \ldots, Z^p(X^{P*})\}$.

Step 3. The initial fuzzy model is given by the aspiration level with each objective as follows:
Find x_{ijk} i =1,2,3,…,m, j =1,2,3,...,n and k=1,2,3,...,l, so as to satisfy $Z^p \leq L^p$ where p = 1, 2, 3,…,P, and the given constraints and non-negativity conditions.

Step 4. For the multi-objective interval solid transportation, a membership function $\mu^p(Z^p)$ corresponding to p-th criterion is defined as

$$\mu^p(Z^p) = \begin{cases} 1 & \text{if } Z^p \leq L^p \\ \left(\dfrac{U^p - Z^p}{U^p - L^p}\right) & \text{If } L^p < Z^p < U^p \\ 0 & \text{if } Z^p \geq U^p, \end{cases}$$

where $U^p \neq L^p$ forall p. If $U^p = L^p$ for all p then $\mu^p(Z^p) = 1$ for all p.

Step 5. Formulate a fuzzy linear programming problem. By using max-min operator, the equivalent fuzzy linear programming problem for the multi-objective interval solid transportation problem is formulated as follows:

For right limit of the objective function $Z_R^p(x)$, the fuzzy linear programming problem is obtained as

Maximize λ

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\{c_{Cijk}^{p}+c_{Wijk}^{p}\}x_{ijk} + \lambda(U^p - L^p) \le U^p,\ p = 1, 2, 3,...,P,$$

with the given constraints and $\lambda \ge 0$, where $\lambda = \min\{\mu^p(Z^p)\}$.

For the centre of the objective function $Z_C^p(x)$, the fuzzy linear programming problem is obtained as

Maximize λ

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}x_{ijk}+\lambda(U^p - L^p) \le U^p,\ p = 1, 2, 3,...,P,$$

with the given constraints and $\lambda \ge 0$, where

$$\lambda = \min\{\mu^p(Z^p)\}.$$

Find out an optimal solution of the foregoing problem by using any existing method. Substituting this optimal value in each objective we get an optimal compromise interval of each objective.

8. Numerical Example:

When the objective function coefficients c_{ijk}^{p}, source, destination and conveyance parameters a_i, b_j and e_k are in the form of stochastic intervals, the MOISTP can be formulated as:

Minimize

$$Z^p = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}[c_{Lijk}^{p}, c_{Rijk}^{p}]\,x_{ijk},$$

$$p = 1, 2, 3,..., P$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = [a_{Li}, a_{Ri}],$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = [b_{Lj}, b_{Rj}],$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = [e_{Lk}, e_{Rk}],$$ with

$$\sum_{i=1}^{m} a_{Li} \ge \sum_{j=1}^{n} b_{Lj},\ \sum_{i=1}^{m} a_{Ri} \ge \sum_{j=1}^{n} b_{Rj},\ \sum_{k=1}^{l} e_{Lk} \ge \sum_{j=1}^{n} b_{Lj},\ \sum_{k=1}^{l} e_{Rk} \ge \sum_{j=1}^{n} b_{Rj}$$

(non-balanced condition). Where $x_{ijk} \ge 0$ for any $i = 1, 2, 3, \ldots, m$, $j = 1, 2, 3, \ldots, n$ and $k = 1, 2, 3, \ldots, l$, $[c_{Lijk}^{1}, c_{Rijk}^{1}]$ and $[c_{Lijk}^{2}, c_{Rijk}^{2}]$ are interval cost matrices for the criterians 1 and 2 respectively (Tables-1 ,2,3,4).

Using equations (16) and (17) and the expectation of random variables, the MOISTP is equivalent to

minimize

$$Z_R^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}[c_{Cijk}^{p}+c_{Wijk}^{p}]x_{ijk},$$

$$Z_C^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}[c_{Cijk}^{p}]x_{ijk},$$

where $p = 1, 2, 3, \ldots, P$,

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \ge E[a_{Li}],$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le E[a_{Ri}],$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge E[b_{Lj}],$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \le E[b_{Rj}],$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \ge E[e_{Lk}],$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le E[e_{Rk}],$$

where $x_{ijk} \ge 0$, for any $i = 1, 2, 3, \ldots, m$, $j = 1, 2, 3, \ldots, n$ and $k = 1, 2, 3, \ldots, l$. The following numerical example illustrates the solution procedure of the foregoing problem.

Example 3. Minimize

$$Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}[c_{Lijk}^{1}, c_{Rijk}^{1}]\,x_{ijk},$$

$$Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2}[c_{Lijk}^{2}, c_{Rijk}^{2}]\,x_{ijk},$$

subject to

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} = [N(32,5), N(90, 7)],$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} = [N(40, 5), N(95,7)],$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{3jk} = [N(36,7), N(98, 4)],$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{i1k} = [\mathcal{EXP}(20), \mathcal{EXP}(35)],$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{i2k} = [\mathcal{EXP}(15), \mathcal{EXP}(43)],$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{i3k} = [\mathcal{EXP}(20), \mathcal{EXP}(40)],$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij1} = [U(25, 65), U(36, 80)],$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij2} = [U(23,50), U(60, 80)].$$

where $x_{ijk} \geq 0$, for i , j = 1,2, 3, k = 1, 2.

The equivalent deterministic MOISTP can be expressed as:

minimize

$$Z_R^1(x)= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2} [c_{Rijk}^1]x_{ijk},$$

$$Z_R^2(x)= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2} [c_{Rijk}^2]x_{ijk},$$

$$Z_C^1(x) = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2} [c_{Cijk}^1]x_{ijk},$$

$$Z_C^2(x) = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{2} [c_{Cijk}^2]x_{ijk},$$

subject to

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \geq 32, \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} \leq 90, \sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} \geq 40, \sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} \leq 95,$$

$$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{3jk} \geq 36, \sum_{j=1}^{3}\sum_{k=1}^{2} x_{3jk} \leq 98,$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{i1k} \geq 20, \sum_{i=1}^{3}\sum_{k=1}^{2} x_{i1k} \leq 35,$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{i2k} \geq 15, \sum_{i=1}^{3}\sum_{k=1}^{2} x_{i2k} \leq 43,$$

$$\sum_{i=1}^{3}\sum_{k=1}^{2} x_{i3k} \geq 20, \sum_{i=1}^{3}\sum_{k=1}^{2} x_{i3k} \leq 40,$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij1} \geq 45, \sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij1} \leq 58,$$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij2} \geq 36.5, \sum_{i=1}^{3}\sum_{j=1}^{3} x_{ij2\backslash} \leq 70.$$

where $x_{ijk} \geq 0$,for i , j = 1, 2, 3, k = 1, 2,

c_{Rijk}^1, c_{Rijk}^2 c_{Cijk}^2 and c_{Cijk}^2 are given as follows. Using fuzzy approach, the pareto optimal solution of the problem is obtained as x_{111}=19.0415, x_{131}=12.9585, x_{211}=5.9585, x_{221}=7.0, x_{232}=27.0415, x_{321}=0.0415, x_{322}=35.9565, λ = 0.5634 and other x_{ijk} are zeros.

Z^1 = [680.4324, 1287.403] and

Z^2 = [671.7095, 1240.917].

9. Conclusion: This paper proposes a solution procedure for multi-objective interval solid transportation problem under stochastic environment using fuzzy programming approach. All source availability, destination demand and conveyance capacities have been taken as stochastic intervals for each criterion. Expectation of a random variable has been used to transform the problem into a classical multi-objective transportation problem where the objectives which are the right limit and centre of the interval objective functions are minimized. The main advantage of fuzzy programming is that, for a MOISTP with 'p' objective functions, this approach leads to p non-dominated solutions and one optimal compromise solution, whereas other algorithms leads to more than p non-dominated and dominated solutions from which the decision maker can choose a compromise solution.

Table–1. Interval cost matrix for first criterion consisting of 3 sources, 3 destinations and 2 conveyances .

	j = 1		j = 2		j = 3	
i = 1	[N(7, 2), N(15,1)]		[N(5, 1), N(13, 4)]		[N(7, 1), N(12, 2)]	
		[N(9, 4), N(12, 1)]		[N(7, 2), N(22, 1)]		[N(9, 4), N(14, 2)]
i = 2	[N(10, 2), N(14, 3)]		[N(7, 2), N(12, 3)]		[N(7, 2), N(17, 1)]	
		[N(9, 3), N(12, 4)]		[N(5, 1), N(25, 4)]		[N(5, 1), N(9, 2)]
i = 3	[N(9, 2), N(24, 2)]		[N(6, 3), N(15, 4)]		[N(6, 2), N(15, 3)]	
		[N(5, 2), N(25, 4)]		[N(6,3), N(12, 3)]		[N(5,4), N(23,2)]

	j = 1		j = 2		j = 3	
i = 1	[N(6, 2), N(14,1)]		[N(4, 1), N(14, 4)]		[N(8, 1), N(13, 2)]	
		[N(7, 2), N(14, 3)]		[N(6,1), N(20, 2)]		[N(9, 2), N(15, 3)]
i = 2	[N(9, 2), N(15, 4)]		[N(7, 3), N(11, 3)]		[N(5, 1), N(16, 2)]	
		[N(8, 2), N(12, 3)]		[N(6, 1), N(23, 2)]		[N(5, 1), N(9, 2)]
i = 3	[N(8, 1), N(22, 4)]		[N(5, 1), N(14, 2)]		[N(6,1), N(14, 2)]	
		[N(5, 2), N(24, 2)]		[N(6,2), N(11, 3)]		[N(7,1), N(21,3)]

Table–2. Interval cost matrix for second criterion consisting of 3 sources, 3 destinations and conveyances .

	j = 1		*j = 2*		*j = 3*	
i =1	*N(14,1)*		*N(4, 1)*		*N(8, 1)*	
		N(7, 2)		*N(16,1)*		*N(9, 2)*
i =2	*N(15,4)*		*N(7, 3)*		*N(15, 1)*	
		N(8, 2)		*N(16, 2)*		*N(9, 1)*
i =3	*N(8, 1)*		*N(15, 2)*		*N(16,1)*	
		N(11, 2)		*N(6,2)*		*N(7,1)*

Table – 3. c^{1}_{ijk} transportation cost for first criterion

	j = 1		*j = 2*		*j = 3*	
i =1	*N(14,1)*		*N(4, 1)*		*N(8, 1)*	
		N(7, 2)		*N(16,1)*		*N(9, 2)*
i =2	*N(15,4)*		*N(7, 3)*		*N(15, 1)*	
		N(8, 2)		*N(16, 2)*		*N(9, 1)*
i =3	*N(8, 1)*		*N(15, 2)*		*N(16,1)*	
		N(11, 2)		*N(6,2)*		*N(7,1)*

Table- 4. c^{2}_{ijk} transportation cost for second criterion.

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