The Strong non Split Domination Number of Fuzzy Graphs

C.Y.Ponnappan[1], S. Basheer Ahamed[2], P. Surulinathan[3]

[1] Department of Mathematics , Government Arts College Paramakudi , Tamilnadu, India
[2] Department of Mathematics , P.S.N.A. College of Engineering and Technology, Dindigul, Tamilnadu, India.
[3]Department of Mathematics , Lathamathavan Engineering college, Kidaripatti, Alagarkovil, Madurai-625301,Tamilnadu,India.

Abstract – A dominating set D of a fuzzy graph G=(σ,μ) is a Strong non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ′,μ′) is complete. The strong non split domination number $\gamma_{sns}(G)$ of G is the minimum fuzzy cardinality of a strong non split dominating set. In this paper we study a strong non split dominating sets of fuzzy graphs and investigate the relationship of $\gamma_{sns(G)}$ with other known parameter of G.

Keywords – Fuzzy graphs , Fuzzy domination ,Split fuzzy domination number , Strong Split fuzzy domination number, strong non split domination number.

I. Introduction

Kulli V.R. et.al introduced the concept of split domination and non-split domination in graphs [3]. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness[10]. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs[11]. Mahyoub Q.M. and Sonar N.D. discussed the split domination number of fuzzy graphs [6]. In this paper we discuss the strong non split domination number of fuzzy graph and obtained the relationship with other known parameter of G.

II. Preliminaries

Definition:2.1 [2]

Let G=(V,E) be a graph. A subset D of V is called a dominating set in G if every vertex in V-D is adjacent to some vertex in D. The domination number of G is the minimum cardinatliy taken over all dominating sets in G and is denoted by $\gamma(G)$.

Definition:2.2 [2]

A dominating set D of a fuzzy graph G=(σ,μ) is connected dominating set if the induced fuzzy sub graph H=(<D>,σ′,μ′) is connected. The minimum fuzzy cardinality of a connected dominating set of G is called the connected dominating number of G and is denoted by $\gamma_c(G)$.

Definition:2.3 [3]

A dominating set D of a graph G=(V,E) is a split dominating set if the induced subgraph <V-D> is disconnected. The split domination number $\gamma_s(G)$ of a graph G is the minimum cardinality of a split dominating set.

Definition:2.4 [3]

A dominating set D of a graph G=(V,E) is a non split dominating set if the induced subgraph <V-D> is connected. The non split domination number $\gamma_{ns}(G)$ of a graph G is the minimum cardinality of a non split dominating set.

Definition:2.5 [4]

A dominating set D of a graph G=(V,E) is a strong non split dominating set if the induced subgraph <V-D> is complete. The strong non split domination number $\gamma_{sns}(G)$ of a graph G is the minimum cardinality of a strong non split dominating set.

Definition : 2.6 [10]

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy graph G=(σ,μ) is a set with two functions $\sigma : V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(\{u ,v\}) \le \sigma(u) \wedge \sigma(v)$ for all $u,v \in V$.

Definition : 2.7 [11]

Let G=(σ,μ) be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(u)=\sigma(u)$ for all $u \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(\{u ,v\}) = \mu(\{u ,v\})$ for all $u,v \in V_1$, then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $<V_1>$.

Definition : 2.8 [11]

The fuzzy subgraph $H=(V_1,\sigma_1,\mu_1)$ is said to be a spanning fuzzy subgraph of G=(V,σ,μ) if $\sigma_1(u)=\sigma(u)$ for all $u \in V_1$ and $\mu_1(u,v) \le \mu(u,v)$ for all $u,v \in V$. Let G

(V,σ,μ) be a fuzzy graph and σ_1 be any fuzzy subset of μ, i.e. , $\sigma_1(u) \le \sigma(u)$ for all u.

Definition : 2.9 [6]

A dominating set D of a fuzzy graph G=(σ,μ) is a split dominating set if the induced fuzzy subgraph H=(<V-D>,σ′,μ′) is disconnected.

The split domination number $\gamma_s(G)$ of G is the minimum fuzzy cardinality of a split dominating set.

Definition : 2.10 [6]

A dominating set D of a fuzzy graph G=(σ,μ) is a non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ',μ') is connected.

The non split domination number $\gamma_{ns}(G)$ of G is the minimum fuzzy cardinality of a non split dominating set.

Definition : 2.11

A dominating set D of a fuzzy graph G=(σ,μ) is a strong non split dominating set if the induced fuzzy subgraph H=(<V-D>,σ',μ') is complete.

The strong non split domination number $\gamma_{sns}(G)$ is the minimum fuzzy cardinality of a strong non split dominating set.

Definition : 2.12 [11]

The order p and size q of a fuzzy graph G=(σ,μ) are defined to be $p=\sum_{u\in V}\sigma(u)$ and $q=\sum_{(u,v)\in E}\mu(\{u,v\})$.

Definition : 2.13 [11]

An edge e={u ,v} of a fuzzy graph is called an effective edge if $\mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)$.

$N(u) = \{ v\in V / \mu(\{u,v\}) = \sigma(u) \wedge \sigma(v)\}$ is called the neighborhood of u and $N[u]=N(u) \cup \{u\}$ is the closed neighborhood of u.

The effective degree of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by dE(u). $\sum_{v\in N(u)} \sigma(v)$ is called the neighborhood degree of u and is denoted by dN(u). The minimum effective degree $\delta_E(G)=\min\{dE(u)|u\in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u)|u\in V(G)\}$.

Definition : 2.14 [11]

The complement of a fuzzy graph G denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma,\bar{\mu})$ where $\bar{\mu}(\{u,v\}) = \sigma(u)\wedge \sigma(v) - \mu(\{u,v\})$.

Definition : 2.15 [11]

Let σ:V→[0,1] be a fuzzy subset of V. Then the complete fuzzy graph on σ is defined to be (σ,μ) where $\mu(\{u,v\})=\sigma(u)\wedge\sigma(v)$ for all uv∈E and is denoted by K_σ.

Definition : 2.16 [11]

A fuzzy graph G=(σ,μ) is said to be bipartite if the vertex V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Further if $\mu(u,v)=\sigma(u) \wedge \sigma(v)$ for all $u\in V_1$ and $v\in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.

Definition : 2.17 [11]

A dominating set D of a fuzzy graph G is said to be a minimal dominating if no proper subset D′ of D is dominating set of G such that |D′|<|D|.

Proposition : 1

For fuzzy bipartite graph K_{σ_1,σ_2}, $\gamma_{sns}(K_{\sigma_1,\sigma_2}) = 0$.

Proposition : 2

If the fuzzy graph $G=2K_2$ with equal membership for all vertices and edges then $\gamma(\bar{G})=\gamma_s(\bar{G}) = \gamma_{ss}(\bar{G}) = \gamma_{ns}(\bar{G})= \gamma_{sns}(\bar{G})$

Proposition : 3

For any fuzzy path, $\gamma_{sns}(P_p)=0$.

Proposition : 4

For any fuzzy cycle, $\gamma_{sns}(C_p) = 0$

Theorem : 1

For any fuzzy graph G=(σ,μ) $\gamma(G) \le \gamma_{sns}(G)$.

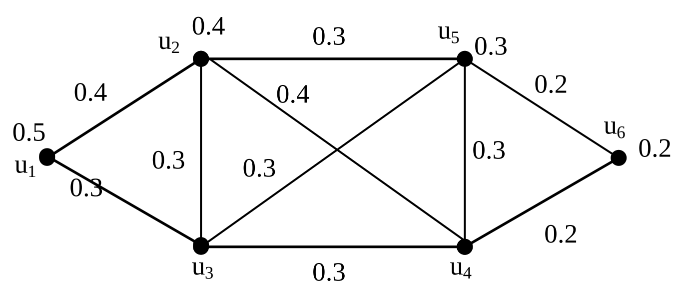
Proof

Let G=(σ,μ) be a fuzzy graph. Let D be the minimum dominating set. D_{sns} is the fuzzy strong non split dominating set. D_{sns} is also a dominating set but need not be a minimum fuzzy dominating set.

Therefore we get $|D|\le|D_{sns}|$

That is $\gamma(G) \le \gamma_{sns}(G)$.

Example



0.3 Fig. (i) 0.6

Here D = $\{u_3,u_5\}$ $D_{sns} = \{u_1,u_6\}$

$\gamma(G) = 0.6$ $\gamma_{sns}(G) = 0.7$

Theorem : 2

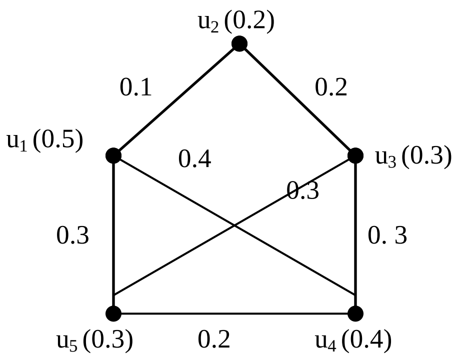
For any fuzzy graph G=(σ,μ), $\gamma(G) \leq \min\{\gamma_s(G), \gamma_{sns}(G)\}$

Proof :

Let G=(σ,μ) be a fuzzy graph. D be the minimum fuzzy dominating set. Let D_s and D_{sns} the minimum fuzzy split dominating set and minimum fuzzy strong non split dominating set of G respectively. The cardinality of fuzzy dominating set need not exceeds either one of the minimum of cardinality of fuzzy split dominating set or fuzzy strong non split dominating set.

Therefore $|D| \leq \min\{|D_s|, |D_{sns}|\}$
Hence $\gamma(G) \leq \min\{\gamma_s(G), \gamma_{sns}(G)\}$

Example :



$D=\{u_3,u_5\}$, $\gamma(G) = 0.6$
$D_s=\{u_1,u_3\}$, $\gamma_s(G) = 0.8$
$D_{sns}=\{u_1,u_2\}$, $\gamma_{sns}(G) = 0.7$

Theorem : 3

For any spanning fuzzy sub graph $H = (\sigma', \mu')$ of G=(σ,μ),

$$\gamma_{sns}(H) \geq \gamma_{sns}(G)$$

Proof

Let G=(σ,μ) be a fuzzy graph and let H(σ′,μ′) be the fuzzy spanning sub graph of G. $D_{sns}(G)$ be the fuzzy minimum strong non-split dominating set of G. $D_{sns}(G)$ is fuzzy strong non-split dominating set of H but not minimum.

Therefore $\gamma_{sns}(H) \geq \gamma_{sns}(G)$.

Example

Spanning fuzzy sub graph H of G (Fig (i))

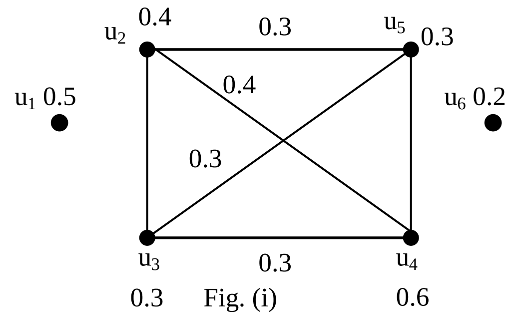


Fig. (i)

$\gamma_{sns}(G) = 0.7$, $\gamma_{sns}(H) = 1.0$

Theorem : 4

For any complete fuzzy graph K_σ then $\gamma(G) = \gamma_{sns}(G) = \min\{\sigma(u)/u \in V\}$

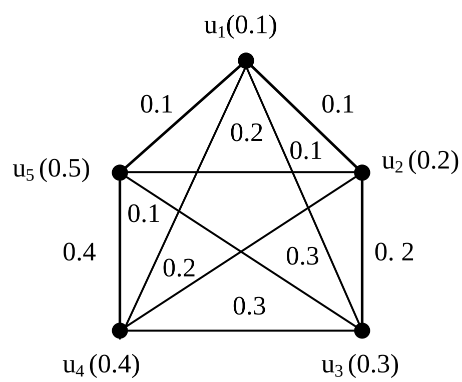
Proof

Let G=(σ,μ) be a complete fuzzy graph therefore there is a strong arc between every pair of vertices. We remove any vertex having minimum cardinality, the resulting graph is complete.

Let {v} is minimum dominating set then <V-D> is complete.
Therefore,

$$\gamma(G) = \gamma_{sns}(G) = \min\{\sigma(u)/u \in V\}$$

Example :



$\gamma(G) = \gamma_{sns}(G) = 0.1$

Theorem : 5

A strong non split dominating set D of G=(σ,μ) is minimal if and only if for each v∈D one of the following two conditions holds

(i) N(v)∩D=φ
(ii) There is a vertex u∈V-D
Such that N(u)∩D={v}

Proof :

Let D be a minimal strong non split dominating set and $v \in D$, then $D'=D-\{v\}$ is not a strong non-split dominating set and hence there exist $u \in V-D'$ such that u is not dominated by any element of D'. If u=v we get (i) and if $u \neq v$ we get (ii). The converse is obvious.

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