Intuitionistic Fuzzy Multi–Objective Structural Optimization using Non-linear Membership Functions

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Abstract
In this paper, we develop an intuitionistic fuzzy optimization approach using non-linear membership and non-membership function for optimizing multi objective structural model. In this optimum design formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. A classical truss optimization example is given to demonstrate the efficiency of the Intuitionistic fuzzy optimization approach with non-linear membership function. A three-bar planar truss subjected to a single load condition is considered as a test problem. Numerical example is given to illustrate our approach.

Keywords - *Structural Design, Intuitionistic fuzzy optimization, Non-linear membership and non-membership function*.

I. INTRODUCTION

In the context of structural design the uncertainty is connected with lack of accurate data of design factors. This problem has been solving by use of fuzzy mathematical algorithm for dealing with this class of problems. However the problem may be an optimization problem where one or more constraints are simultaneously satisfied subject to the minimization of the weight function. Again sometimes it does not hold good in real world problems where multiple and conflicting objectives frequently exist. The accomplishment of this task is due to the methodology known as multi-objective structural optimization (MOSO). The MOSO is gaining importance especially in the last decade due to the increasing technological demand of structural optimization.

Bellman[14] and Zadeh [11] incorporate the fuzzy set theory to the decision making problem. The fuzzy set theory also found application in Structural design. Several researchers like Huang et.al [6], Wang et al. [16], Rao [13] ,Yeh et al. [18], Xu [17], Shih et.al [4], Dey et. al [5] ,have distinctive contribution to fuzzy set theory as well as fuzzy optimization.

Atanassov[3,8-10] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory characterized by a membership function, a non-membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. The concept of membership and non-membership was first considered by Angelov[1] in optimization problem and gave intuitionistic fuzzy approach to solve this. Luo.et al. [19] applied the inclusion degree of intuitionistic fuzzy set to multi criteria decision making problem. Pramanik et al.[12] solved a vector optimization problem using an intuitionistic fuzzy goal programming. A transportation model was solved by Jana et al.[7] using multi-objective intuitionistic fuzzy linear programming. Dey et al. [15] use Intuitionistic fuzzy optimization technique to optimize non-linear single objective two bar truss structural model.

In this paper, a well-known three bar truss design model is considered as a Structural design model. The results are compared numerically with both in fuzzyoptimization technique and intuitionistic fuzzy optimization techniquefor non-linear membership function. From our numerical result, it is clear that intuitionistic fuzzy optimization provides better results than fuzzy optimization.

The motivation of the present study is to give computational algorithm for solving multi-objective nonlinearprogramming problem by Intuitionistic fuzzy optimization approach and the impact of various type of membership functions in computation of Intuitionistic fuzzy optimization and thus made comparative study of linear and nonlinear membership.

The remainder of this paper is organized in the following way. In section II, we discuss about Multi-objective Structural Model. In section III, we discuss about fuzzy set, intuitionistic fuzzy set, α-cut and β-cuts. In section IV, we discuss Solution of Multi-objective Nonlinear Programming Problem by fuzzy and intuitionistic Fuzzy Non-Linear Programming technique with linear membership and non-membership functions. In section V, we discuss about Solution of Multi-objective structural optimization Problem by fuzzy and intuitionistic fuzzy optimization technique. In section VI, we discuss about numerical solution of structural model of three bar truss and compared results by Intuitionistic Fuzzy Non-linear programming (IFNLP) technique and by fuzzy non-linear programming (FNLP) technique. Finally we draw conclusions from the results in section VII.

II. MULTI-OBJECTIVE STRUCTURAL MODEL

In the design problem of the structure i.e lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure .In truss structure system,the basic parameters (including allowable stress etc.) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions .

The multi-objective structural model can be expressed as

$$Minimize\ WT(A) \tag{1}$$

$$Minimize\ \delta(A)$$

$$subject\ to\ \sigma(A) \le [\sigma]$$

$$A^{\min} \le A \le A^{\max}$$

Where $A = [A_1, A_2, ..., A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar ,$WT(A) = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure , $\delta(A)$ is the deflection of the loaded joint ,where L_i, A_i and ρ_i are the bar length ,cross section area and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is allowable stress of the group bars under various conditions, $A^{\min}$ and $A^{\max}$ are the lower and upper bounds of cross section area A respectively.

III. MATHEMATICAL PRELIMINARIES

A. Fuzzy Set

The entire Let X denotes a universal set. Then the fuzzy subset A in X is a subset of order pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0,1]$ is called the membership function which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$ to each element $x \in X$. A is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set. It is clear that the range of membership function is a subset of non-negative real numbers.

B. $\alpha-$ Level set or $\alpha-$ Cut of a Fuzzy Set

The $\alpha-$level set of a fuzzy set A of X is a crisp set A_α which contains all the elements of X that have membership values greater than or equal to α i.e $A = \{x : \mu_A(x) \ge \alpha, x \in X, \alpha \in [0,1]\}$.

C. Intuitionistic Fuzzy Set

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) set A^i in the sense of Attanassove [14] is given by equation $A^i = \{< X, \mu_{A^i}(x), \upsilon_{A^i}(x) > | x_i \in X\}$ where the function $\mu_{A^i}(x^i) : X \to [0,1]$; $x_i \in X \to \mu_{A^i}(x_i) \in [0,1]$ and $\upsilon_{A^i}(x^i) : X \to [0,1]; x_i \in X \to \upsilon_{A^i}(x_i) \in [0,1]$. define the degree of membership and degree of non-membership of an element $x_i \in X$ to the set $A^i \subseteq X$,such that they satisfy the condition $0 \le \mu_{A^i}(x_i) + \upsilon_{A^i}(x_i) \le 1$, $\forall\ x_i \in X$. For each IFS A^i in X the amount $\Pi_{A^i}(x_i) = 1 - (\mu_{A^i}(x^i) + \upsilon_{A^i}(x^i))$ is called the degree of uncertainty (or hesitation) associated with the membership of elements $x_i \in X$ in A^i we call it intuitionistic fuzzy index of A^i with respect of an element.

D. (α, β) Level Intervals or (α, β) cuts

A set of $(\alpha, \beta)-$cut, generated by an IFS A^i where $(\alpha, \beta) \in [0,1]$ are fixed number such that $\alpha + \beta \le 1$ is defined as

$$A^i_{\alpha,\beta} = \left\{ \begin{array}{c} < x, \mu_{A^i}(x), \upsilon_{A^i}(x) > / x \in X \\ \mu_{A^i}(x) \ge \alpha, \upsilon_{A^i}(x) \le \beta, \alpha, \beta \in [0,1] \end{array} \right\}$$

.We define $(\alpha, \beta)-$ level or $(\alpha, \beta)-$ cut ,denoted by $A^i_{\alpha,\beta}$,as the crisp set of elements x which belong to A^i at least to the degree α and which belong to A^i at most to the degree α .

IV. MATHEMATICAL ANALYSIS

A. Fuzzy Non-linear Programming (FNLP) Technique to Solve Multi-objective non-linear Programming Problem Document

A multi-objective non-linear programming (MONLP) Problem may be taken in the following form

$$Minimize\ \{f_1(x), f_2(x), ..., f_p(x)\}^T \tag{2}$$

$$\text{Subject to } g_j(x) \le b_j; \qquad j = 1, 2, ..., m$$

$$x > 0$$

Following Zimmermann [4] ,we have presented a solution algorithm to solve the MONLP Problem by fuzzy optimization technique.

Step-1: Solve the MONLP (2) as a single objective non-linear programming problem p th by taking one of the objective at a time and ignoring the others .These solutions areknown as ideal solutions. Let x^i be the respective optimal solution for the i^{th} different objectives with same constraints and evaluate each objective values for all these i^{th} optimal solutions.

Step-2: From the result of step -1 determine the corresponding values for every objective for each derived solutions.With the values of all objectives at each ideal solutions ,pay-off matrix can be formulated as follows

$$\begin{array}{cc} & \begin{array}{cccc} f_1(x) & f_2(x) & \cdots\cdots\cdots & f_p(x) \end{array} \\ \begin{array}{c} x^1 \\ x^2 \\ \vdots \\ x^p \end{array} & \begin{bmatrix} f_1^*(x^1) & f_2^*(x^1) & \cdots\cdots\cdots & f_p^*(x^1) \\ f_1^*(x^2) & f_2^*(x^2) & \cdots\cdots\cdots & f_p^*(x^2) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ f_1^*(x^p) & f_2^*(x^p) & \cdots\cdots\cdots & f_p^*(x^p) \end{bmatrix} \end{array}$$

Here $x^1, x^2, ..., x^p$ are the ideal solution of the objectives $f_1(x), f_2(x), ..., f_p(x)$ respectively.

Step-3: From the result of step-2,now we find lower bound (minimum) L_i and upper bound (maximum) U_i by using the following rule $U_i = \max\{f_i(x_p)\}, L_i = \min\{f_i(x_p)\}$ where $1 \le i \le p$ .It is obvious $L_i = f_i^*(x^i),\ 1 \le i \le p$.

Step-4: Using aspiration level of each objective, the MONLP (2)maybe written as follows

Find x so as to satisfy (3)

$$f_i(x) \le L_i \quad (i = 1, 2, ..., p)$$

$$g_j(x) \le b_j; \quad j = 1, 2, ..., m$$

$$x > 0$$

Here objective function of (2) are consider as fuzzy constraints. This type of fuzzy constraint can be quantified by eliciting a corresponding membership function $\mu_i(f_i(x)),\ i = 1, 2, ..., p$

$$\mu_i(f_i(x)) = \begin{cases} 1 & \text{if } f_i(x) \le L_i^{ACC} \\ \dfrac{e^{-w\left(\frac{f_i(x) - L_i^{ACC}}{U_i^{ACC} - L_i^{ACC}}\right)} - e^{-w}}{1 - e^{-w}} & \text{if } L_i^{ACC} \le f_i(x) \le U_i^{ACC} \\ 0 & \text{if } f_i(x) \ge U_i^{ACC} \end{cases}$$

Under the concept of mean operator, the feasible solution set is defined by intersection of the fuzzy objective set .The feasible set is then characterized by its membership $\mu_D(x)$ which is

$$\mu_D(x) = \min\{\mu_1(f_1(x)), \mu_2(f_2(x)), ..., \mu_p(f_p(x))\} \quad (4)$$

The decision solution can be obtained by solving the problem of *maximize* $\mu_D(x)$ subject to the given constraints i.e

$$\underset{\forall x > 0}{Maximize}\left(\underset{\forall i}{Minimize}\ \mu_i(x)\right) \quad (5)$$

$$such\ that\ g_j(x) \le b_j,$$

$$x > 0, \quad j = 1, 2, ..., m, i = 1, 2, ..., p$$

Now if suppose $\alpha = Minimize\, \mu_i(x)$ be the overall satisfactory level of compromise, then we obtain the following equivalent model

$$Maximize\ \ \alpha \quad (6)$$

$$such\ that\ \mu_i(x) \ge \alpha, \quad i = 1, 2, ..., p$$

$$g_j(x) \le b_j, \quad j = 1, 2, ..., m$$

$$x > 0, \quad \alpha \in [0, 1]$$

Step-5: Solve (7) to get optimal solution.

B. An Intuitionistic Fuzzy Approach for Solving Multi-Objective Non-Linear Programming Problem with Non-linear membership and Non-linear Non-membeship Function

Following Zimmermann [20] and Angelov [2],we have presented a solution algorithm to solve MONLP (2) by Intuitionistic fuzzy optimization (IFO). Here Step 1 and Step 2 are same as shown in (IV.A)

Step-3: From the result of step 2 now we find lower bound (minimum) L_i^{ACC} and upper bound (maximum) U_i^{ACC} by using following rule $U_i^{ACC} = \max\{f_i(x^p)\}, L_i^{ACC} = \min\{f_i(x^p)\}$ where $1 \le i \le p$ .But in IFO The degree of non-membership (rejection) and the degree of membership (acceptance) are considered so that the sum of both value is less than one. To define the non-membership of NLP problem let $U_i^{\mathrm{Re}\,j}$ and $L_i^{\mathrm{Re}\,j}$ be the upper bound and lower bound of objective function $f_i(x)$ where $L_i^{ACC} \le L_i^{\mathrm{Re}\,j} \le U_i^{\mathrm{Re}\,j} \le U_i^{ACC}$.

For objective function of minimization problem ,the upper bound for non-membership function (rejection) is always equals to that the upper bound of membership function (acceptance).One can take lower bound for non-membership function as follows $L_i^{\mathrm{Re}\,j} = L_i^{Acc} + \varepsilon_i$ where

$0 < \varepsilon_i < \left(U_i^{Acc} - L_i^{Acc}\right)$ based on the decision maker choice.

The initial intuitionistic fuzzy model with aspiration level of objectives becomes $Find\ \{x_i, i = 1, 2, ..., p\}$

so as to satisfy $f_i(x) \le^i L_i^{Acc}$ with tolerance $P_i^{Acc} = \left(U_i^{Acc} - L_i^{Acc}\right)$ for the degree of acceptance for i=1,2,…,p. $f_i(x) \ge^i U_i^{\mathrm{Re}\,j}$ with tolerance $P_i^{Acc} = \left(U_i^{Acc} - L_i^{Acc}\right)$ for degree of rejection for $i = 1,2,...,p$ .Define the membership (acceptance) and non-membership (rejection) functions of above uncertain objectives as follows. For the $i^{th}, i = 1,2,...,p$ objectives functions the linear membership function $\mu_i\left(f_i(x)\right)$ and linear non-membership $\upsilon_i\left(f_i(x)\right)$ is defined as follows

$$\mu_i\left(f_i(x)\right) = \begin{cases} 1 & if \quad f_i(x) \le L_i^{Acc} \\ \dfrac{e^{-w\left(\frac{f_i(x) - L_i^{Acc}}{U_i^{Acc} - L_i^{Acc}}\right)} - e^{-w}}{1 - e^{-w}} & if \quad L_i^{Acc} \le f_i(x) \le U_i^{Acc} \\ 0 & if \quad f_i \ge U_i^{Acc} \end{cases}$$

$$\upsilon_i\left(f_i(x)\right) = \begin{cases} 0 & if \quad f_i(x) \le L_i^{\mathrm{Re}\,j} \\ \left(\dfrac{f_i(x) - L_i^{\mathrm{Re}\,j}}{U_i^{\mathrm{Re}\,j} - L_i^{\mathrm{Re}\,j}}\right)^2 & if \quad L_i^{\mathrm{Re}\,j} \le f_i(x) \le U_i^{\mathrm{Re}\,j} \\ 1 & if \quad f_i(x) \ge U_i^{\mathrm{Re}\,j} \end{cases}$$

Step-4: Now an Intuitionistic fuzzy optimization for above problem with membership and non-membership function can be written as

$$\underset{\forall i}{Maximize} \left(\mu_i\left(f_i(x)\right)\right) \tag{7}$$

$$\underset{\forall i}{Minimize} \left(\upsilon_i\left(f_i(x)\right)\right)$$

$$subject\ to\ \mu_i\left(f_i(x)\right) + \upsilon_i\left(f_i(x)\right) < 1$$
$$\left(\mu_i\left(f_i(x)\right)\right) > \left(\upsilon_i\left(f_i(x)\right)\right);$$
$$\left(\upsilon_i\left(f_i(x)\right)\right) \ge 0;$$
$$g_j(x) \le 0;$$
$$x > 0$$
$$i = 1,2,...,p;\ j = 1,2,...,m$$

Find an equivalent crisp model by using membership and non-membership functions of objectives by IF as follows

$$Max\left(Min\left(\mu_1, \mu_2, .., \mu_p\right)\right) - Min\left(Max\left(\upsilon_1, \upsilon_2, ..., \upsilon_p\right)\right)$$

$$subject\ to\ \mu_i\left(f_i(x)\right) + \upsilon_i\left(f_i(x)\right) < 1 \tag{8}$$
$$\left(\mu_i\left(f_i(x)\right)\right) > \left(\upsilon_i\left(f_i(x)\right)\right);$$
$$\left(\upsilon_i\left(f_i(x)\right)\right) \ge 0;$$
$$g_j(x) \le 0;$$
$$x > 0$$
$$i = 1,2,...,p;\ j = 1,2,...,m$$

If we consider $\alpha = Minimize\left(\mu_1, \mu_2, ..., \mu_p\right)$; $\beta = Maximize\left(\upsilon_1, \upsilon_2, ..., \upsilon_p\right)$ accordingly the Angelov [15], the above can be written as

$$Maximize\ \ (\alpha - \beta) \tag{9}$$

$$subject\ to\ \mu_i\left(f_i(x)\right) \ge \alpha;$$
$$g_j(x) \le 0;$$
$$x > 0, \alpha + \beta \le 1$$
$$\alpha \in [0,1], \beta \in [0,1];\ i = 1,2,...,p$$
$$j = 1,2,................,m$$

which on substitution of $\mu_i\left(f_i(x)\right)$ *and* $\upsilon_i\left(f_i(x)\right)$ *for* $i = 1,2,...,p$ becomes

$$Maximize\ (\alpha - \beta) \tag{10}$$

$$subject\ to$$
$$f_i(x) + \frac{U_i^{Acc} - L_i^{Acc}}{w} \ln\left\{\left(1 - e^{-w}\right)\alpha + e^{-w}\right\} \le L_i^{Acc};$$
$$f_i(x) - \sqrt{\beta}\left(U_i^{\mathrm{Re}\,j} - L_i^{\mathrm{Re}\,j}\right) \le L_i^{\mathrm{Re}\,j};$$
$$g_j(x) \le 0;$$
$$\alpha + \beta \le 1;$$
$$\alpha \in [0,1], \beta \in [0,1]$$
$$i = 1,2,...,p;\ j = 1,2,...,m$$

Step-5: Solve the above crisp model (11) by using appropriate mathematical programming algorithm to get optimal solution of objective function.

Step-6: Stop.

V. SOLUTION OF MULTI-OBJECTIVE STRUCTURAL OPTIMIZATION PROBLEM BY FUZZY AND INTUITIONISTIC FUZZY OPTIMIZATION TECHNIQUE

To solve the MOSOP (1) step 1 of IV is used. After that according to step 2 pay-off matrix is formulated

$$\begin{array}{c c} & \begin{array}{cc} WT(A) & \delta(A) \end{array} \\ \begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} WT^*\left(A^1\right) & \delta\left(A^1\right) \\ WT\left(A^2\right) & \delta^*\left(A^2\right) \end{bmatrix} \end{array}$$

In next step following step 2 we calculate the bound of the objective U_1^{Acc}, L_1^{Acc} and $U_1^{\mathrm{Re}\,j}, L_1^{\mathrm{Re}\,j}$ for weight function $WT(A)$,such that $L_1^{Acc} < WT(A) < U_1^{Acc}$ and $L_2^{\mathrm{Re}\,j} < WT(A) < U_2^{\mathrm{Re}\,j}$ and $U_2^{Acc}, L_2^{Acc}; U_2^{\mathrm{Re}\,j}, L_2^{\mathrm{Re}\,j}$ for deflection $\delta(A)$, such that $L_2^{Acc} < WT(A) < U_2^{Acc}$ and $L_2^{\mathrm{Re}\,j} < \delta(A) < U_2^{\mathrm{Re}\,j}$ with the condition $U_i^{Acc} = U_i^{\mathrm{Re}\,j};\ \ L_i^{\mathrm{Re}\,j} = L_i^{Acc} + \varepsilon_i\ \ for\ i = 1,2$ so as $0 < \varepsilon_i < \left(U_i^{Acc} - L_i^{Acc}\right)$ are identified.

According to IFO technique considering membership and non-membership function for MOSOP (1)

$$\mu_{WT(A)}\left(WT(A)\right)=\begin{cases}1 & \text{if } WT(A)\le L_{WT}^{Acc}\\ \dfrac{e^{-w\left(\frac{WT(A)-L_{WT}^{Acc}}{U_{WT}^{Acc}-L_{WT}^{Acc}}\right)}-e^{-w}}{1-e^{-w}} & \text{if } L_{WT}^{Acc}\le WT(A)\le U_{WT}^{Acc}\\ 0 & \text{if } WT(A)\ge U_{WT}^{Acc}\end{cases}$$

$$\upsilon_{WT(A)}\left(WT(A)\right)=\begin{cases}0 & \text{if } WT(A)\le L_{WT}^{\mathrm{Re}\,j}\\ \left(\dfrac{WT(A)-L_{WT}^{\mathrm{Re}\,j}}{U_{WT}^{\mathrm{Re}\,j}-L_{WT}^{\mathrm{Re}\,j}}\right)^2 & \text{if } L_{WT}^{\mathrm{Re}\,j}\le WT(A)\le U_{WT}^{\mathrm{Re}\,j}\\ 1 & \text{if } WT(A)\ge U_{WT}^{\mathrm{Re}\,j}\end{cases}$$

And

$$\mu_{\delta(A)}\left(\delta(A)\right)=\begin{cases}1 & \text{if } \delta(A)\le L_{\delta}^{Acc}\\ \dfrac{e^{-w\left(\frac{\delta(A)-L_{\delta}^{Acc}}{U_{\delta}^{Acc}-L_{\delta}^{Acc}}\right)}-e^{-w}}{1-e^{-w}} & \text{if } L_{\delta}^{Acc}\le \delta(A)\le U_{\delta}^{Acc}\\ 0 & \text{if } \delta(A)\ge U_{\delta}^{Acc}\end{cases}$$

$$\upsilon_{\delta(A)}\left(\delta(A)\right)=\begin{cases}0 & \text{if } \delta(A)\le L_{\delta}^{\mathrm{Re}\,j}\\ \left(\dfrac{\delta(A)-L_{\delta}^{\mathrm{Re}\,j}}{U_{\delta}^{\mathrm{Re}\,j}-L_{\delta}^{\mathrm{Re}\,j}}\right)^2 & \text{if } L_{\delta}^{\mathrm{Re}\,j}\le \delta(A)\le U_{\delta}^{\mathrm{Re}\,j}\\ 1 & \text{if } \delta(A)\ge U_{\delta}^{\mathrm{Re}\,j}\end{cases}$$

crisp non-linear programming problem is formulated as follows

$$\begin{aligned}&Max\left(Min\left(\mu_{WT}\left(WT(A)\right)\right),\mu_{\delta}\left(\delta(A)\right)\right)\\ &\quad -Min\left(Max\left(\upsilon_{WT}\left(WT(A)\right)\right),\upsilon_{\delta}\left(\delta(A)\right)\right)\end{aligned} \tag{11}$$

subject to

$$\mu_{WT}\left(WT(A)\right)+\upsilon_{WT}\left(WT(A)\right)<1;$$
$$\mu_{\delta}\left(\delta(A)\right)+\upsilon_{\delta}\left(\delta(A)\right)<1;$$
$$\mu_{WT}\left(WT(A)\right)>\upsilon_{WT}\left(WT(A)\right);$$
$$\mu_{\delta}\left(\delta(A)\right)>\upsilon_{\delta}\left(\delta(A)\right);$$
$$\mu_{WT}\left(WT(A)\right)\ge 0,\upsilon_{WT}\left(WT(A)\right)\ge 0;$$
$$\mu_{\delta}\left(\delta(A)\right)\ge 0,\upsilon_{\delta}\left(\delta(A)\right)\ge 0;$$
$$\sigma[A]\le[\sigma];A>0$$

According to Angelov [2] the above problem can be written as

$$Maximize\ (\alpha-\beta) \tag{12}$$

subject to

$$\mu_{WT}\left(WT(A)\right)\ge\alpha;\ \upsilon_{WT}\left(WT(A)\right)\le\beta;$$
$$\mu_{\delta}\left(\delta(A)\right)\ge\alpha;\ \upsilon_{\delta}\left(\delta(A)\right)\le\beta;$$
$$\sigma[A]\le[\sigma],\ \alpha+\beta\le 1;$$
$$A>0,\alpha\in[0,1],\beta\in[0,1]$$

Solve the above crisp model (13) by an appropriate mathematical programming algorithm to get optimal solution and hence objective function i.e structural weight and deflection of loaded joint will get the Pareto optimal solution.

VI. NUMERICAL ILLUSTRATION

A well-known three bar planer truss is considered is to minimize weight $WT(A_1,A_2)$ of the structure and minimize the deflection $\delta(A_1,A_2)$ at a loading point of a statistically loaded three bar planer truss subject to stress constraints on each of the truss members

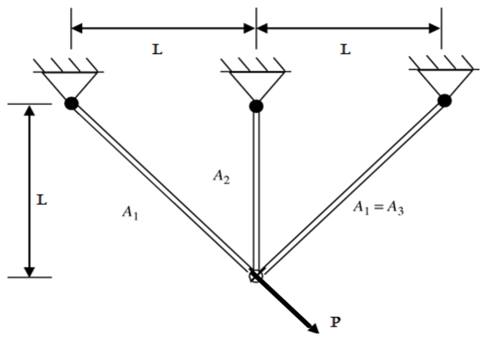

Fig.1. Design of Three Bar Planer Truss

The multi-objective optimization problem can be stated as follows

$$Minimize\ \ WT(A_1,A_2)=\rho L\left(2\sqrt{2}A_1+A_2\right) \tag{13}$$

$$Minimize\ \ \delta(A_1,A_2)=\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)}$$

$$subject\ \ to\ \ \sigma_1(A_1,A_2)\equiv\frac{P\left(\sqrt{2}A_1+A_2\right)}{\left(2A_1^2+2A_1A_2\right)}\le\left[\sigma_1^T\right];$$

$$\sigma_2(A_1,A_2)\equiv\frac{P}{\left(A_1+\sqrt{2}A_2\right)}\le\left[\sigma_2^T\right];$$

$$\sigma_3(A_1,A_2)\equiv\frac{PA_2}{\left(2A_1^2+2A_1A_2\right)}\le\left[\sigma_3^C\right];$$

$$A_i^{\min}\le A_i\le A_i^{\max}\quad i=1,2$$

where $P=$ applied load ; $\rho=$ material density ; $L=$ length ; $E=$ Young's modulus ; A_1 = Cross section of bar-1 and bar-3; A_2 = Cross section of bar-2; δ is deflection of loaded joint. $\left[\sigma_1^T\right]$ and $\left[\sigma_2^T\right]$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, $\left[\sigma_3^C\right]$ is maximum allowable compressive stress for bar 3.

Solution: According to step 2 pay-off matrix is formulated as follows

$$\begin{array}{c c}
 & \begin{array}{cc} WT(A_1,A_2) & \delta(A_1,A_2) \end{array}\\
\begin{array}{c} A^1 \\ A^2 \end{array} & \begin{bmatrix} 2.638958 & 14.64102 \\ 19.14214 & 1.656854 \end{bmatrix}
\end{array}$$

Here
$U_{WT}^{\upsilon} = U_{WT}^{\mu} = 19.14214, L_{WT}^{\upsilon} = L_{WT}^{\mu} + \varepsilon_1 = 2.638958 + \varepsilon_1;$
Such that $0 < \varepsilon_1 < (19.14214 - 2.638958)$;
$U_{\delta}^{\upsilon} = U_{\delta}^{\mu} = 14.64102, L_{\delta}^{\upsilon} = L_{\delta}^{\mu} + \varepsilon_2 = 1.656854 + \varepsilon_2;$
such that $0 < \varepsilon_2 < (14.64102 - 1.656854)$.
Here truth, indeterminacy, and falsity membership function for objective functions are $WT(A_1A_2), \delta(A_1A_2)$ are defined as follows

$$\mu_{WT}(WT(A_1,A_2)) = \begin{cases} 1 & if\ WT(A_1,A_2) \le 2.638958 \\ \dfrac{e^{-w\left(\frac{WT(A_1,A_2)-2.638958}{16.503182}\right)} - e^{-w}}{1-e^{-w}} & if\ 2.638958 \le WT(A_1,A_2) \le 19.14214 \\ 0 & if\ WT(A_1,A_2) \ge 19.14214 \end{cases}$$

$$\upsilon_{WT}(WT(A_1,A_2)) = \begin{cases} 0 & if\ WT(A_1,A_2) \le 2.638958+\varepsilon_1 \\ \left(\dfrac{WT(A_1,A_2)-(2.638958+\varepsilon_1)}{16.503182-\varepsilon_1}\right)^2 & if\ 2.638958+\varepsilon_1 \le WT(A_1,A_2) \le 19.14214 \\ 1 & if\ WT(A_1,A_2) \ge 19.14214 \end{cases}$$

$$\mu_{\delta}(\delta(A_1,A_2)) = \begin{cases} 1 & if\ \delta(A_1,A_2) \le 3.638958 \\ \dfrac{e^{-w\left(\frac{\delta(A_1,A_2)-3.638958}{11.002062}\right)} - e^{-w}}{1-e^{-w}} & if\ 3.638958 \le \delta(A_1,A_2) \le 14.64102 \\ 0 & if\ \delta(A_1,A_2) \ge 14.64102 \end{cases}$$

$$\upsilon_{\delta}(\delta(A_1,A_2)) = \begin{cases} 0 & if\ \delta(A_1,A_2) \le 3.638958+\varepsilon_2 \\ \left(\dfrac{\delta(A_1,A_2)-(3.638958+\varepsilon_2)}{11.002062-\varepsilon_2}\right)^2 & if\ 3.638958+\varepsilon_2 \le \delta(A_1,A_2) \le 14.64102 \\ 1 & if\ \delta(A_1,A_2) \ge 14.64102 \end{cases}$$

Now using IFO technique with membership and non-membership functions we get

$$Maximize\ (\alpha - \beta) \tag{14}$$

subject to

$$\left(2\sqrt{2}A_1 + A_2\right) + \frac{16.503182}{w}\ln\left\{e^{-w} + \left(1-e^{-w}\right)\alpha\right\} \le 2.638958;$$

$$\frac{20}{\left(\sqrt{2}A_2 + A_1\right)} + \frac{11.002062}{w}\ln\left\{e^{-w} + \left(1-e^{-w}\right)\alpha\right\} \le 3.638958;$$

$$(2\sqrt{2}A_1 + A_2) - \sqrt{\beta}\,(16.503182 - \varepsilon_1) \le (2.638958 + \varepsilon_1);$$

$$\frac{20}{\left(\sqrt{2}A_2 + A_1\right)} - \sqrt{\beta}\,(11.002062 - \varepsilon_2) \le (3.638958 + \varepsilon_2);$$

$$\frac{20\left(\sqrt{2}A_1 + A_2\right)}{\left(2A_1^2 + 2A_1A_2\right)} \le 20;$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \le 20;$$

$$\frac{20A_2}{\left(2A_1^2 + 2A_1A_2\right)} \le 15;$$

$$\alpha \in [0,1], \beta \in [0,1];$$

$$w = 2;\ 0.1 \le A_1, A_2 \le 5;$$

The Pareto optimal solution of MOSOP (14) using fuzzy and Intuitionistic fuzzy multi objective non-linear programming technique is given in table 2. Here we get best solution for different tolerance ε_1 and ε_2 for non linear membership and non-membership function of IFO method. From Table-2 it shows that Non-linear IFO gives better result.

Table.1 The Input Data for MOSOP (13)

Applied Load P (KN)	Material Density (ρ) (KN/m^3)	Length L (m)	Maximum allowable tensile stress $[\sigma_T]$ (KN/m^2)	Maximum allowable compressive stress $[\sigma_C]$ (KN/m^2)	Young's Modulus E (KN/m^2)	$A_i^{\min}$ and $A_i^{\max}$ of cross section of bars $10^{-4}(m^2)$
20	100	1	20	15	2×10^8	$A_i^{\min} = 0.1$, $A_i^{\max} = 5, i = 1,2.$

Table 2. Comparisons of Optimal Solution of MOSOP (14) Based on Different Method

Method	$A_1 \times 10^{-4} m^2$	$A_2 \times 10^{-4} m^2$	$WT \times 10^2\, KN$	$\delta \times 10^{-7} m$
Intuitionistic Fuzzy Non-linear Programming (IFNLP)using linear membership function $\varepsilon_1 = 1.6503182, \varepsilon_2 = 1.1002062$	0.5766526	3.694181	5.325201	3.447673
Intuitionistic Fuzzy Non-linear Programming (IFNLP) using non-linear membership function $\varepsilon_1 = 1.6503182, \varepsilon_2 = 1.1002062$	0.5513085	2.634761	4.194097	4.675712

VII. CONCLUSIONS

In view of comparing the intuitionistic fuzzy optimization with fuzzy optimization method for membership and non-membership we also obtained the solution of the undertaken numerical problem by fuzzy optimization method given by Zimmermann and intuitionistic fuzzy optimization method given by Angelov. The main objective of this work is to illustrate the impact of nonlinear membership and non-membership of IFO technique in utilization of nonlinear structural problem . Here we have considered a non-linear three bar truss design problem .In this problem, we find out optimum weight of the structure in presence of optimum deflection of loaded joint. The comparison of results obtained for the undertaken problem clearly show the difference between the linear and non-linearintuitionistic fuzzy optimization in perspective of structural design. The results of this study may lead to the development of effective non linear IFO i.e (NLIFO) technique solving other nonlinear model in different field.

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