Intuitionistic Fuzzy Basis in Topological Spaces

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Abstract

In this paper, the notion of intuitionistic fuzzy basis and strong intuitionistic fuzzy basis is introduced. Theorems related to crisp basis, intuitionistic fuzzy basis and strong intuitionistic fuzzy basis are stated and proved.

I. INTRODUCTION

In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets. The notion of defining intuitionistic fuzzy sets (IFSs) for fuzzy set generalizations, introduced by Atanassov[1], has proven interesting and useful in various application areas. Since this fuzzy set generalization can present the degrees of membership and non-membership with a degree of hesitancy, the knowledge and semantic representation becomes more meaningful and applicable. Chang (1968) [3] was the first to introduce the concept of a fuzzy topology. Muthukumari et al., initiated the concept of fuzzy basis. A.M.Ali et al., introduced the concept of intuitionistic fuzzy sequence in metric spaces.

In this paper, the concept of intuitionistic fuzzy basis and strong intuitionistic fuzzy basis is introduced. Some results regarding to crisp basis, intuitionistic fuzzy basis and strong intuitionistic fuzzy basis are investigated.

II. PRELIMINARIES

Definition 2.1

Let $X = \{x_1, x_2, \dots, x_n\}$ be a non-empty finite set A fuzzy set F of X can be defined as $F = \{(x, F(x))/ x \in X\}$;

where $F(x): X \to [0, 1]$ is the degree of membership of x in X.

Definition 2.2

A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:

1. $\varphi, X \in T$,
2. If $A, B \in T$ then $A \cap B \in T$
3. If $A_i \in T$ for each $i \in I$, then $\cup_{i \in I} \in A_i \in T$ The pair (X , T) is called a fuzzy topological space.

Definition 2.3

Let X be a non empty set.

$B \subset P(X)$ is called a base if

1.$\cup\{B / B \in \mathbf{B}\} = X$.

2.$U, V \in \mathbf{B}$ and $x \in U \cap V$ implies there exists $W \in \mathbf{B}$ such that $x \in W \subset U \cap V$.

Let T be the collection of all union of finite number of elements of **B**.

Then T is a topology and **B** is a base for the topology.

Definition 2.4

Let X be a nonempty set. A function $f: P(X) \to [0,1]$ is called a fuzzy basis if

1.$\cup\{B / f(B) = 1\} = X$.

2. For each $\alpha \in (0,1]$. $f(U) \geq \alpha$, $f(V) \geq \alpha$ and $x \in U \cap V$ implies there exist W with $f(W) \geq \alpha$ and $x \in W \subset U \cap V$.

Definition 2.5

Let X be a nonempty set. $\mathbf{B} \subset P(X)$ is called a strong crisp basis if

1.$\cup\{B / B \in \mathbf{B}\} = X$.

2.$U, V \in B$, $U \cap V \neq \varphi \Rightarrow U \cap V \in \mathbf{B}$.

Example 2.1

1.$X = \{a, b, c\}$, $\mathbf{B} = \{\{a, b\}, \{b, c\}, \{b\}\}$

2.$X = \{a, b, c, d\}$, $\mathbf{B} = \{a, b, c\}, \{b, c, d\}, \{b, c\}\}$.

Definition 2.6

An intuitionistic fuzzy set F in X can be formulated as $F = \{(x, \mu_F(x), \nu_F(x))/x \in X\}$ where $\mu_F(x), \nu_F(x): X \to [0,1]$ represent the degree of membership and non-membership of x in X, respectively, with the essential condition $0 \leq \mu_F(x) + \nu_F(x) \leq 1$.

III. INTUITIONISTIC FUZZY BASIS

Definition 3.1

Let X be a nonempty set. A function (f_μ, f_ν) : $P(X) \to [0,1]$ is called an intuitionistic fuzzy basis if

1.$\cup\{B/f_\mu(B) = 1, f_\nu(B) = 0\} = X$,

2.For each $(\alpha, \beta) \in (0,1]$ with $\alpha + \beta \leq 1$, $f_\mu(U) \geq \alpha$, $f_\nu(U) \leq \beta$ and $f_\mu(V) \geq \alpha$, $f_\nu(V) \leq \beta$ and $x \in U \cap V$ implies there exist W with $f_\mu(W) \geq \alpha$, $f_\nu(W) \leq \beta$ and $x \in W \subset U \cap V$.

Example 3.1

Let $= \{a, b, c, d\}$.$(f_\mu, f_\nu): P(X) \to [0,1]$ as

f_μ (X) = 1, f_γ (X) = 0,
f_μ { a ,b ,c } = 1, f_γ {a ,b ,c } = 0,
f_μ { b, c, d} = 1, f_γ {b ,c ,d } = 0,
f_μ { b} = 1, f_γ { b} = 0,
f_μ { c} = 1, f_γ { c} = 0,
f_μ {a ,b ,d } = 0.6, f_γ {a ,b ,d } = 0.2,
f_μ { a, b, c} = 0.6, f_γ { a, b, c} = 0.3,
f_μ { α} = 0.4, f_γ { α } = 0.1,
f_μ {d } = 0.5, f_γ {d } = 0.2, and
f_μ (A) = 0, f_γ (A) = 1 for all other A ⊂ X,
f is a fuzzy basis. Take α = 0.7,β = 0.1.
Take U= {a ,b ,c }, V= { b, c, d}, f_μ (U) ≥ α, f_γ (U) ≤ β and f_μ (V) ≥ α, f_γ (V) ≤ β ,
b∈ U∩V . Take W= { b }. b∈ W ⊂ U∩V and f_μ (W) = f_γ {B } = 1 ≥ α , f_γ (W) = f_γ {B } = 0 ≤ β . Hence (f_μ , f_γ) is an intuitionistic fuzzy basis.

Definition 3.2
(Strong Intuitionistic Fuzzy Basis)
Let X be a nonempty set. A function (f_μ , f_γ) :P(X) → [0,1]is called a strong fuzzy basis if
1.∪{B/ f_μ(B)=1,V f_γ (B) = 0}=X
2. f_μ (U∩V) ≥ min {f_μ (U), f_μ (V) } and
f_γ (U∩V) ≤ max { (f_γ (U), f_γ (V)}for U,V ⊂ X with U∩V ≠ φ.

Example 3.2
Let X= {a ,b ,c ,d }. (f_μ , f_γ) :P(X) → [0,1]as
f_μ { X} = 1, f_γ { X} = 0,
f_μ { a, b, c} = 0.5, f_γ { a, b, c} = 0.2,
f_μ { b, c, d} = 0.6, f_γ { b, c, d} = 0.3,
f_μ { a, b} = 0.5, f_γ { a, b} = 0.3,
f_μ {A } = 0.5, f_γ {A} = 1 for all other A
U { (B/ f_μ(B) = 1, f_γ (B = 0}=X
Take U = { a, b, c}, V= {a ,b ,d },
U ∩V = {a ,b }, f_μ (U ∩V) = 0.5,
f_γ (U ∩V) = 0.2
min{ f_μ (U), f_μ (V)} = {0.5,0.6} = 0.5,
max { f_μ (U), f_μ (V)} = {0.2,0.3} = 0.3
Now, f_μ (U ∩V) ≥ min{ f_μ (U), f_μ (V)} and
f_γ (U ∩V) ≤ max{ f_γ (U), f_γ (V)}.
Hence (f_μ , f_γ) is a strong intuitionistic fuzzy basis.

Theorem 3.1
Every strong intuitionistic fuzzy basis is an intuitionistic fuzzy basis.
Proof:
Let X be a nonempty set. Let f_μ , f_γ:P (X) → [0,1] be a strong intuitionistic fuzzy basis. Then
1. U { (B/ f_μ(B) = 1, f_γ (B = 0}=X
2. f_μ (U ∩V) ≥ min{ f_μ (U), f_μ (V)} and
f_γ (U ∩V) ≤ max{ f_γ (U), f_γ (V)} for U,V⊂ X with U∩V ≠ φ.
Claim: (f_μ , f_γ) is an intuitionistic fuzzy basis. (1) U { (B/ f_μ(B) = 1, f_γ (B = 0}=X .
(2). Let α,β∈(0,1] with α+ β + ≤ 1. Let f_μ (U) ≥ α , f_γ (U) ≤ β and f_μ (V) ≥ α , f_γ (V) ≤ β and
x∈ U∩V. Now take W = U∩V . Now x∈W⊂U∩V . Also f_μ (W) = f_μ (U∩V) ≥ min { f_μ (U), f_μ (V)} ≥ α and f_γ (W) = f_γ (U∩V) ≤ max { f_γ (U), f_γ (V)} ≤ β So f_μ (W) ≥ α, f_γ (W) ≥ β . Hence there exist W
such that x ∈ W ⊂ U ∩V and f_μ (W) ≥ α,
f_γ (W) ≥ β. Hence (f_μ , f_γ) is an intuitionistic fuzzy basis.

Remark 3.1
The converse of the above theorem is not true.

Example 3.3
Let X= {a ,b ,c ,d }.Define (f_μ , f_γ) :P(X) → [0,1]as
f_μ { a, b, c,d} =1, f_γ { a, b, c,d} = 0,
f_μ {a, b, c} = 0.6, f_γ { a,b, c} = 0.4,
f_μ {b,c,d} = 0.6, f_γ {b,c,d} = 0.3,
f_μ {b} = 0.6, f_γ {b} = 0.2,
f_μ {c} = 0.6, f_γ {c} = 0.1,
f_μ {A} = 0, f_γ {A} = 1 for all otherA.
The conditions (a) and (b) satisfied. Hence is an intuitionistic fuzzy basis.
U= {a ,b ,c },V = {b ,c ,d },U∩V = {b ,c },
$f_{\mu(}$U∩V) = f_μ(b,c) = 0.min{f_μ(U), f_μ(V)} = min(0.6.0.6) = 0.6.
f_γ $_($U∩V) = f_γ(b,c) = 0.max {f_γ (U), f_γ (V)}= max(0.4.0.3)=0.4.
f_μ(U∩V) is not greater than or equal to min{ f_μ (U), f_μ (V }.
f_γ(U∩V) is not less than or equal to max { f_γ (U), f_γ (V)}.
Hence f is not a strong intuitionistic fuzzy basis.

Theorem 3.2
Every crisp basis induces an intuitionistic fuzzy basis.
Proof:
Let X be a non empty set. Let **B** be a crisp basis. Define f:P (X) → [0,1] as f(A) = 1 if A∈ **B** and f(A) = 0 if A doesnot belong to **B**
(1) ∪ { B / (B) = 1} =∪ { B / B∈ **B** } = X , by definition of crisp basis.
(2) Take α ,β ∈ (0,1] with α+β ≤ 1. Let f_μ (U) ≥ α , f_γ (U) ≤ β and x ∈ U∩V . f_μ (U) ≥ α ,
f_γ (U) ≤ β and f_μ (V) ≥ α , f_γ (v) ≤ β implies f_μ (U) = 1, f_γ (U) = 0 and f_μ (V) = 1, f_γ (V) =0. This implies that U,V ∈ **B.** Since **B** is a crisp basis ∃ W ∈ **B**x ∈ W ⊂ U∩V . Since W ∈ **B, f**(W) = 1 and hence f_μ (W) ≥ α , f_γ (W) ≤ β
Hence ∃ W such that x∈ W ⊂U ∩V and f_μ (U) ≥ α , f_γ (U) ≤ β.Hence f is an intuitionistic
fuzzy basis. Hence every crisp basis induces an intuitionistic fuzzy basis.

Theorem 3.3
The fuzzy basis induced by a strong crisp basis is a strong fuzzy basis.
Proof:
Let X be a non empty set. Let **B** be a strong crisp basis. Let f= (f_μ , f_γ) be the induced fuzzy
basis. We claim that f= (f_μ , f_γ) is a strong fuzzy basis. Take U,V ∈P(X) where U∩V ≠ φ. If f_μ (U) =

0, f_ν (U) = 1 f_μ (V) = 1, f_ν (V) = 0 then whatever be the value of (U ∩V), we have f_μ(U∩V) ≥ min{ f_μ (U), f_μ (V }≤ max { f_ν (U), f_ν (V)}.If f_μ (U) = 1, f_ν (U) = 0 or f_μ (V) = 1, f_ν (V) = 0 then U,V∈ **B**. Since **B** is a strong basis, U∩V ∩ ∈ **B .**

Hence f_μ(U∩V) ≥ 1, f_ν (U∩V) ≤ 0.

Hence f_μ(U∩V) ≥ min{ f_μ (U), f_μ (V }≤ max { f_ν (U), f_ν (V)}.

Hence f is a strong intuitionistic fuzzy basis

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