Dependent chance goal programming model for multi-objective interval solid transportation problem in stochastic environment

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***Abstract**:- In this paper Dependent Chance Goal Programming Model (DCGPM) is mainly focused on the classical Multi-Objective Solid Transportation Problem (MOSTP), where the multiple interval objective functions are minimized and the order relations that represent the decision makers' preference between interval profits have been defined by the right limit, left limit, centre and half width of an interval. Finally, the equivalent transformed problem has been modeled in this paper, so that it could be solved by using a evolutionary technique.*

Keywords: *Multi-objective solid transportation problem; Multi-objective interval solid transportation problem; Solid transportation problem; Stochastic programming; Goal programming; Dependent chance goal programming*

1.INTRODUCTION

The Solid transportation problem (STP) in uncertain environment becomes important branch of optimization and a lot of models and algorithms have been presented for different problems by different authors, Liu[3, 4], Deshpande et al. [5], Papadrakakis and Lagaros [6], Lixing Yang and Linzhong Liu [7] and so on. Lixing Yang and Yuan Feng [8] presented three types of models in stochastic environment namely expected value goal programming model, chance-constrained goal programming model and dependent-chance goal programming model. S.R.Arora and Archana KHURANA. [9], designed and developed an algorithm for solid fixed charge bi-criterion indefinite quadratic transportation problem. A.Nagarajan and K.Jeyaraman developed many models and methods for solid fixed charge bi-criterion indefinite quadratic transportation problem and MOISTP under stochastic environment[30, 31, 32, 33, 34]. S.K.Das et al. [29], developed the theory and methodology for multi-objective transportation problem with interval cost, source and destination parameters.

In this paper, by using the idea of stochastic environment, dependent chance goal programming model for MOISTP has been proposed, in which the coefficients of the objective functions are taken in the form of stochastic intervals. Using some methodologies, an equivalent crisp model to the given MOISTP has been constructed and in order to illustrate the modeling the numerical examples are provided.

This paper is organized as follows. In Section 2, the basic idea of MOSTP and MOISTP has been given. In Section 3, definitions of interval arithmetic and related definitions have been given. In Section 4, the formulation of crisp objective function is given. Goal programming models for MOISTP is given in Section 5. Chance-constrained goal programming model and dependent-chance goal programming model are constructed in Section 6. In Section 7, several crisp equivalences for different models have been investigated. Numerical example is provided in Section 8.

2. PRELIMINARIES

It is well known that the MOSTP involves in transporting homogeneous products from 'm' sources to 'n' destinations by 'k' conveyances so that the total transportation cost is minimized. In this paper, the basic knowledge of MOISTP in stochastic environment is considered.

2.1 MULTI-OBJECTIVE INTERVAL SOLID TRANSPORTATION PROBLEM

A homogeneous product is to be transported from 'm' sources to 'n' destinations. The sources are production facilities, warehouses, or supply points, that are characterized by available capacities a_i, i = 1, 2, 3,…,m. The destinations are consumption facilities, warehouses, or demand points that are characterized by required levels of demand b_j, j = 1, 2, 3,…,n. Let e_k, for k = 1, 2, 3,…,l be the amount of this product which can be carried by 'l' different modes of transport called conveyances, such as trucks, air freight, goods trains, ship, etc.. A penalty or cost $c^{p}_{ijk} \geq 0$ is associated with transportation of a unit product from source 'i' to destination 'j' by means of the conveyance 'k' for the p^{th} criterion. The penalty cost could represent transportation cost, delivery time, quantity of goods delivered, duty paid, under used capacity, etc., One must determine the amount of product (unknown quantity) x_{ijk} to be transported from all source 'i' to all destinations 'j' by means of each conveyance 'k' such that the total transportation cost is minimized. In the real world, STP are not all single objective, instead considering more objectives in a STP. In a balanced STP, the sum of supplies, the sum of demands and the sum of conveyance capacities are supposed to be equal to each other. But in the real world problems, the balanced condition need not hold always. It is assumed that there are enough products in sources 'm' to satisfy the demands of 'n' destinations, also the conveyances 'l' which have abilities to transport products to satisfy the demand of each destination. Hence for the non-balanced STP, it is

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \quad \sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} b_j.$$

Thus the Multi-Objective Solid Transportation Problem (MOSTP) is the problem of minimizing P objective functions. The MOISTP is a generalization of the MOSTP in which input data are expressed as stochastic intervals instead of point values. These types of problems arise only when uncertainty occurs in data and decision makers consider it as more convenient to express it as intervals.

The 'P' interval valued minimizing objective functions, in which the interval appears only in cost but not on the source, destination and conveyance, that can be considered as stochastic variables. MOISTP is formulated as a linear programming problem as follows:

P1:Minimize $Z^{p} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} [c^{p}_{Lijk}, c^{p}_{Rijk}]\, x_{ijk}$ p = 1, 2, 3,..., P (1)

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i, \quad i = 1, 2, 3,\ldots, m \tag{2}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j, \quad j= 1, 2, 3, \ldots, n. \tag{3}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, 2, 3, \ldots, l. \tag{4}$$

$$x_{ijk} \geq 0, \text{ for all } i, j, k.$$

where the superscripts on Z^{p} denote the p^{th} penalty criterion, $[c^{p}_{Lijk}, c^{p}_{Rijk}]$ for p = 1, 2, 3,...,P are intervals representing the uncertain cost $c^{p}_{ijk} \geq 0$ for the p^{th} criterion for the transportation problem; it can represent delivery time, quantity of goods delivered, under used capacity, etc. The parameters $a_i \geq 0$, $b_j \geq 0$, $e_k \geq 0$, $c_{ijk} \geq 0$, i = 1, 2, 3,…, m, j = 1, 2, 3, ..., n, k = 1, 2, 3, …, l are stochastic variables that follow certain distribution and

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \quad \sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} b_j. \text{ (non-balanced condition).}$$

The existence of a feasible solution to STP is guaranteed [4], and a non-degenerated basic feasible solution contains "m + n + l – 2" nonzero values of the variables for all problems. Haley [2] showed the necessary steps to reformulate the problem and described the solution procedure.

3. INTERVAL ARITHMETIC [10, 11]

In this paper, the upper case letters, A, B, etc., denote closed intervals while the lower case letters, i.e, a, b, etc., denote real numbers. The set of all real numbers is denoted by $\Re$. An interval is defined by an ordered pair of elements as:

$$A=[a_L, a_R]=\{a : a_L \leq a \leq a_R, a \in \Re\} \tag{20}$$

where a_L and a_R are, respectively, the left and right limits of A. The interval is also denoted by its centre and width as:

$$A=\langle a_C, a_W \rangle =\{a : a_C - _W \leq a \leq a_C + a_W, a \in \Re\} \tag{21}$$

where $a_C = (a_R + a_L)/2$ and $a_W = (a_R - a_L)/2$ are, respectively, the centre and half width of A.

DEFINITION 3.1

Let $* \in$ (., /, +, -) be a binary operation on the set of real numbers. If A and B are closed intervals, then

$$A * B = \{a * b : a \in A, b \in B\} \quad (22)$$

defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. The interval operations used in this research paper are given below.

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R], \quad (23)$$

$$A + B = \langle a_C, a_W \rangle + \langle b_C, b_W \rangle = \langle a_C + b_C, a_W + b_W \rangle, \quad (24)$$

$$kA = k[a_L, a_R] = [ka_L, ka_R] \quad \text{for } k \geq 0, \quad (25)$$

$$kA = k[a_L, a_R] = [ka_R, ka_L] \quad \text{for } k < 0, \quad (26)$$

$$Ka = k \langle a_C, a_W \rangle = \langle ka_C, |k| a_W \rangle, \quad (27)$$

where 'k' is real number.

3.1 DEFINITIONS OF ORDER RELATIONS BETWEEN INTERVALS

The order relations which represent the decision makers' preference between interval costs are defined for the minimization problems. Let the uncertain costs from two alternatives be represented by intervals 'A' and 'B' respectively. It is assumed that the cost of each alternative is known only to lie in the corresponding interval.

DEFINITION 3.2 The order relation $\leq_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \leq_{LR} B \text{ iff } a_L \leq b_L \text{ and } a_R \leq b_R,$$

$$A <_{LR} B \text{ iff } A \leq_{LR} B \text{ and } A \neq B. \quad (28)$$

This order relation $\leq_{LR}$ represents the decision makers' preference for the alternative with lower minimum cost and maximum cost, i.e., if $A \leq_{LR} B$, then A is preferred to B..

DEFINITION 3.3 [6] The order relation $\leq_{CW}$ between $A = \langle a_C, a_W \rangle$ and $B = \langle b_C, b_W \rangle$ is defined as

$$A \leq_{CW} B \text{ iff } a_C \leq b_C \text{ and } a_W \leq b_W,$$

$$A <_{CW} B \text{ iff } A \leq_{CW} B \text{ and } A \neq B. \quad (29)$$

This order relation $\leq_{CW}$ represents the decision makers' preference for the alternative with lower expected cost and less uncertainty, i.e., if $A \leq_{CW} B$, then A is preferred to B.

4. FORMULATION OF THE CRISP OBJECTIVE FUNCTION

In this section, the formulation of original interval objective function has been made as a crisp one.

DEFINITION 4.1 $x^0 \in S$ is an optimal solution of the problem P1 iff there is no other solution $x \in S$ which satisfies $Z(x) <_{LR} Z(x^0)$ or

$$Z(x) <_{CW} Z(x^0).$$

THEOREM 4.1 It can be proved that

$$A \leq_{RC} B \text{ iff } A \leq_{LR} B \text{ or } A \leq_{CW} B,$$

$$A <_{RC} B \text{ iff } A <_{LR} B \text{ or } A <_{CW} B, \quad (30)$$

where the order relation $\leq_{RC}$ is defined as $A \leq_{RC} B$ iff $a_R \leq b_R$ and $a_C \leq b_C$, $A <_{RC} B$ iff $A \leq_{RC} B$ and $A \neq B$.

Using the theorem 4.1, Definition 4.1 is simplified as follows.

DEFINITION 4.2 $x^0 \in S$ is an optimal solution of the problem P1 iff there is no other solution $x \in S$ which satisfies $Z(x) <_{RC} Z(x^0)$.

The right limit $Z_R^p(x)$ of the interval objective function is derived from the equations (24) and (27) as

$$Z_R^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^{p}\, |x_{ijk}|$$

(31)

where c_{Cijk}^{p} is the centre and c_{Wijk}^{p} is the half width of the coefficient of x_{ijk} in Z^p.

In the case when $x_{ijk} \geq 0$, i = 1, 2, 3,…, m, j = 1, 2, 3, ..., n, k = 1, 2, 3, …, l, $Z_R^p(x)$ is modified as:

$$Z_R^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^{p}\, x_{ijk}.$$

(32)

The centre of the objective function $Z_C^p(x)$ is defined as

$$Z_C^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk}.$$

(33)

The solution set of the Problem P1 defined by Definition 4.2 is also obtained as the Pareto optimal solution of the multi-objective problem as:

P2: Minimize{ Z_R^p, Z_C^p }, p = 1, 2, 3,…,P,

subject to the constraints (6) – (8) where Z_R^p and Z_C^p are as stated as in equations (32) and (33).

5. GOAL PROGRAMMING MODEL FOR MOISTP

The main aim of the model proposed in Section 2.1, the Problem P1 is to minimize the total transportation cost for 'p' (one for each criterion) objective functions subject to the given set of constraints. In literature, many models and methods have been addressed for obtaining the ideal solution of the multi-objective programming and hence this model is dealt by goal programming technique. The Goal Programming (GP) model is one of the well-known multi-objective mathematical programming models. This model allows as to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions. More precisely, the GP designed to find a solution that minimizes the deviations between the achievement level of the objectives and the goals set for them. In the case where the goal is surpassed, the deviation will be positive and in the case of under achievement of the goal, the deviation will be negative. First developed by Charnes et al.[21] and Charnes and Cooper [22] then applied by Lee [23] and Lee and Clayton [24], the GP model gained a great deal popularity and its use has spread in diversified field such as management of banking, water basins, solid waste, marketing, quality control, human resources, transportation and site selection, agriculture and forestry. In order to get a suitable transportation plan satisfying the given targets and to balance the two objectives defined in (31) and (32), the following priority levels are defined.

At the first priority level, the total transportation cost of the right limit of the interval objective function $Z_R^p(x)$ defined by (32) should not exceed the given target C_R^p. Then we have the goal constraint :

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^{p}\, x_{ijk} + c_R^{p-} - c_R^{p+} = C_R^p$$

(34)

where c_R^{p+} is the over utilization of the transportation cost and c_R^{p-} is the under utilization of the transportation cost for the pth criterian in which c_R^{p+} has to be minimized.

At the second priority level, the total transportation cost of the centre of the objective function $Z_C^p(x)$ given in (33) should not exceed the given target C_C^p. Then we have the goal constraint :

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk} + c_C^{p-} - c_C^{p+} = C_C^p$$

(35)

where c_C^{p+} is the over utilization and c_C^{p-} is the under utilization of the transportation cost for the p^{th} criterion in which c_C^{p+} has to be minimized.

Thus goal programming models for the Problem P1 together with the constraints (34) and (35) is defined as follows.

P3: Minimize $\{ c_R^{p+}, \quad c_C^{p+} \}$ (36)

Subject to $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p} x_{ijk} +$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^{p} x_{ijk} + c_R^{p-} - c_R^{p+} = C_R^{p}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p} x_{ijk} + c_C^{p-} - c_C^{p+} = C_C^{p}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le a_i, \qquad i = 1, 2, 3,\ldots, m.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge b_j, \qquad j = 1, 2, 3, \ldots, n.$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e_k, \qquad k = 1, 2, 3, \ldots, l.$$

$x_{ijk} \ge 0$, for all i, j, k and c_R^{p-}, c_R^{p+}, c_C^{p-},

$c_C^{p+} \ge 0$, for p = 1, 2, 3,...,P.

Since all the parameters are stochastic in nature the MOISTP in certain environment becomes a stochastic MOISTP. Thus the solution of the transportation model, becomes a stochastic interval one. In that situation, it is difficult to handle the problem by certain known methods, and hence the probability theory has been used to solve the problems with randomness. To satisfy the requirements of randomness, different types of stochastic programming models have been developed to suit the different purposes. In order to solve this problem, Liu [25] provided a theoretical frame work of stochastic programming called Dependent Chance Programming (DCP) (including dependent chance multi-objective programming and Dependent Chance Goal Programming (DCGP)). Some real and potential applications of DCP have been presented by Liu and Ku [26], Liu[27], Liu and Iwamura [40]. This paper will construct the theoretical framework of DCGP models in stochastic environment.

6. DEPENDENT CHANCE CONSTRAINED GOAL PROGRAMMING MODEL [DCCGPM]

The goal or aspiration levels assigned to the various objectives can be probabilistic and

$Z_C^p \sim N[\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} (x_{ijk}\mu_{C_{Cijk}^p}\}, \{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^2(\sigma^2_{C_{Cijk}^p})\}]$, respectively, and

$$F_{\alpha_i} = \sup\{F|F=\phi_{ai}^{-1}(1-\alpha_i)\},$$

$$F_{\beta_j} = \inf\{F|\ F = \phi_{bj}^{-1}(\beta_j)\},$$

$$F_{\gamma_k} = \sup\{F|F=\phi_{ek}^{-1}(1-\gamma_k)\}.$$

The crisp equivalents of three probability constraints in the above model can be obtained by using the theorems defined earlier. $\phi_1^{-1}(\alpha)$ and $\phi_2^{-1}(\beta)$ can be calculated by using Theorem 7.4 as follows.

$$\phi_1^{-1}(\alpha)=\phi^{-1}(\alpha)[\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^2(\sigma^2_{C_{Cijk}^p} + \sigma^2_{C_{Wijk}^p})]^{1/2}+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}(\mu_{C_{Cijk}^p}+\mu_{C_{Wijk}^p})$$

and

$$\phi_2^{-1}(\beta)=\phi^{-1}(\beta)[\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^2(\sigma^2_{C_{Cijk}^p})]^{1/2}+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}\mu_{C_{Cijk}^p}$$

where where the decision maker does not know its value with complete certainty. The first formulation of the Stochastic Goal Programming (SGP) models go back to the late 1969s with Contini's [20]works. He considers the goal as uncertain variables with a normal distribution. Stancu-Minasian [18] and Stancu-Minasian and Giurgiutiu [19] present a synthesis of methodologies used in multiple objective programming in a stochastic environment. Several other techniques have been proposed to solve the SGP model. The most popular technique is a Chance Constrained Programming (CCP) developed by Charnes and Cooper [15, 16, 17], offers a powerful means of modeling stochastic decision systems with assumption that the stochastic constraints holds at least 'α' of time, where 'α' is referred to as the confidence level provided as an appropriate safety margin by the decision maker. Then Liu [14] generalized the CCP to the case with not only stochastic constraints but also with stochastic objectives. The main idea of chance constrained programming is to optimize the critical value of the objective function under the probability constraints.

DEFINITION 6.1 Let 'ξ' be a random variable, and $\alpha \in (0,1]$. Then $\xi_{\inf}(\alpha) = \inf\{ r \mid \Pr\{\xi \le r\} \ge \alpha\}$ is called α-critical value of ξ [25].

To construct Chance constrained goal programming model (CCGPM) for MOISTP, the following two priority structures are used:

At the first priority level, the total transportation cost for the right limit of the objective function should not exceed the given target 'C_R^p' for 'P' objective functions with the confidence level 'α' and the first goal constraint will be:

$$\Pr[\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^{p}\, x_{ijk}\} - C_R^p \le r_1] \ge \alpha \quad \ldots\ldots(\mathrm{I})$$

in which the 'α'-positive deviation from the target 'C_R^p', defined by

$$c_R^{p+} = \min\{ r_1 \mid \Pr[\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Wijk}^{p}\, x_{ijk}\} - C_R^p \le r_1] \ge \alpha\} \vee 0$$

will be minimized.

At the second priority level, the total transportation cost for the centre of the interval objective function should not exceed the given target 'C_C^p' for 'P' objective functions with the confidence level 'β' and the second goal constraint will be:

$$\Pr[\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk}\} - C_C^p \le r_2] \ge \beta \quad \ldots(\mathrm{II}),$$

in which the 'β'-positive deviation from the target C_C^p, defined by

$$c_C^{p+} = \min\{ r_2 \mid \Pr[\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{Cijk}^{p}\, x_{ijk}\} - C_C^p \le r_2] \ge \beta\} \vee 0$$

will be minimized.

The models developed in the previous sections are constructed under stochastic environment. In order to find the suitable solution for the models, critical value or credibility measure must be calculated. If the stochastic parameters are complex, the computing objective values subject to the constraints becomes a time consuming one. Due to this, it is better to convert the models into their crisp equivalents by using the appropriate probability levels defined by the decision makers.

THEOREM 6.2.1 Suppose that 'ξ' is a random variable with continuous probability distribution function $\phi(x)$, and the function $g(x, \xi) = h(x) - \xi$. Then for any $\alpha \in (0, 1]$, we have $\Pr\{ g(x, \xi) \le 0\} \ge \alpha$ if and only if $h(x) \le F_\alpha$, where

$F_\alpha = \sup\{ F \mid F = \phi^{-1}(1-\alpha)\}$ [17].

THEOREM 6.2.2 Suppose that 'ξ' is a random variable with continuous probability distribution function $\phi(x)$, and the function $g(x, \xi) = h(x) - \xi$. Then for any $\alpha \in (0, 1]$, it becomes

$\Pr\{ g(x, \xi) \ge 0\} \ge \alpha$ if and only if $h(x) \ge F_\alpha$, where $F_\alpha = \inf\{ F \mid F = \phi^{-1}(\alpha)\}$.

THEOREM 6.2.3 Let 'ξ' be a random variable with continuous, strictly increasing probability distribution function $\phi(x)$. The α - critical value of ξ is

$\xi_{\inf}(\alpha) = \phi^{-1}(\alpha)$.

THEOREM 6.2.4 Let 'ξ' be a normally distributed random variable with $\xi \sim N(\mu, \sigma^2)$. Then α - critical value of ξ is $\xi_{\inf}(\alpha) = \sigma\,\phi^{-1}(\alpha) + \mu$, where $\phi(x)$ is the probability distribution function of standard normal distribution N(0, 1).

By using the above theorems, DCCGPM for the Problem **P3** together with the chance constraints (I) and (II) is obtained as follows:

Suppose that c_{Cijk}^p, c_{Wijk}^p are independent normally distributed random variables defined as $c_{Cijk}^p \sim N(\mu_{C_{Cijk}^p}, \sigma^2_{C_{Cijk}^p})$, $c_{Wijk}^p \sim N(\mu_{C_{Wijk}^p}, \sigma^2_{C_{Wijk}^p})$ and a_i, b_j and e_k are random variables with continuous probability distribution functions $\phi_{a_i}(x)$, $\phi_{b_j}(x)$ and $\phi_{c_k}(x)$, respectively, where i = 1, 2, 3,…,m, j = 1, 2, 3,…,n, k = 1, 2, 3,…,l.

P4: Minimize { c_R^{p+} ,c_C^{p+} } subject to:

$$\phi_1^{-1}(\alpha) + c_R^{p-} - c_R^{p+} = C_R^p$$

$$\phi_2^{-1}(\beta) + c_C^{p-} - c_C^{p+} = C_C^p.$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le F_{\alpha_i}, \quad i = 1, 2, 3,\ldots, m.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge F_{\beta_j}, \quad j = 1, 2, 3, \ldots, n$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le F_{\gamma_k}, \quad k = 1, 2, 3, \ldots, l.$$

where $x_{ijk} \ge 0$, for any i, j, k and c_R^{p-}, c_R^{p+}, c_C^{p-}, $c_C^{p+} \ge 0$ for p = 1, 2, 3,…,P.where ϕ_1(x) and ϕ_2(x) is the probability distribution function of the random variables

$Z_R^p \sim N[\{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}(x_{ijk}\mu_{C_{Cijk}^p} + x_{ijk}\mu_{C_{Wijk}^p})\}, \{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}^2(\sigma^2_{C_{Cijk}^p} + \sigma^2_{C_{Wijk}^p})\}]$ 'ϕ' is the probability distribution function of the standard normal distribution.

7. DETERMINATION OF CRISP VALUES

To find the crisp equivalent of the constraints developed in the previous section, the random simulation technique has been employed here. Generally three kinds of probability constraints in Problem P4 are transformed into their crisp equivalents. But, if the probability distribution functions ϕ_{a_i}(x), ϕ_{b_j}(x), and ϕ_{c_k}(x) are complex, it is difficult to do so and hence, the following random simulation has been used to obtain the approximate values of F_{α_i}, F_{β_j} and F_{γ_k}.

Compute F_{α_i} (or F_{γ_k}) by random simulation.

STEP 1. Generate the numbers $n_1, n_2, n_3, \ldots, n_N$ according to the probability distribution function ϕ_{a_i}(x)(or ϕ_{c_k}(x)).

STEP 2. Let F_{α_i} (or F_{γ_k}) be the N'th largest number in { $n_1, n_2, n_3, \ldots, n_N$ }, where

N' = [α_iN] + 1 (or [γ_kN] + 1 or [α_{Ri} N] + 1 or [γ_{Rk} N] + 1 or [β_{Rj} N] + 1).

STEP 3. Return F_{α_i} (or F_{γ_k}).

Compute F_{β_j} by random simulation

STEP 1. Generate the numbers $n_1, n_2, n_3, \ldots, n_N$ according to the probability distribution function ϕ_{b_j}(x)

STEP 2. Let $F_{\beta j}$ be the N'th smallest number in{ n_1, n_2 , n_3 ,, n_N },

where N'=[β_jN] + 1 (or [α_{Li} N] + 1 or [γ_{Lk}N] + 1 or [β_{Lj}N] + 1).

STEP 3. Return F_{β_j}.

After finding the crisp equivalent of the models developed earlier the following steps are used to calculate the minimum value of 'P' objective functions in each of the model as follows:

STEP 1. Solve the multi-objective interval solid transportation problem using, one objective at a time(ignoring all others) subject to the given set of constraints by using any one of the suitable evolutionary technique. Let $X^{1*} = \{x_{ijk}^1\}$, $X^{2*} = \{x_{ijk}^2\}$, $X^{3*} = \{x_{ijk}^3\}$,…, $X^{P*} = \{x_{ijk}^p\}$ be the optimum solutions for P different single objective interval solid transportation problems.

STEP 2. From the results of step1, the values of all the objective functions will be calculated at all these 'P' optimal points. Then a payoff matrix is formed.. The 'X^{P*}''s are the individual optimal solutions and each of these is used to determine the values of other individual objectives. Thus the payoff matrix is developed as follows:

$$\begin{array}{c} \\ Z^1 \\ Z^2 \\ Z^3 \\ \\ Z^p \end{array}\begin{pmatrix} X^{1*} & X^{2*} & \ldots & X^{P*} \\ Z^1(X^{1*}) & Z^1(X^{2*}) & \ldots & Z^1(X^{P*}) \\ Z^2(X^{1*}) & Z^2(X^{2*}) & \ldots & Z^2(X^{P*}) \\ Z^3(X^{1*}) & Z^3(X^{2*}) & \ldots & Z^3(X^{P*}) \\ & & & \\ Z^p(X^{1*}) & Z^p(X^{2*}) & \ldots & Z^p(X^{P*}) \end{pmatrix}$$

STEP 3. From Step 2 we find minimum value of each objective function .

Let it be $Z^1_{\min}$, $Z^2_{\min}$, $Z^3_{\min}$, …, $Z^p_{\min}$.

From the above values, the minimum value of the multi-objective interval solid transportation problem can be taken as min Z^p = minimum of

$$\{ Z^1_{\min}, Z^2_{\min}, Z^3_{\min}, \ldots, Z^p_{\min} \}.$$

8. NUMERICAL EXAMPLES

To illustrate the models developed in this paper, a multi-objective interval solid transportation problem having the following characteristics is considered for Problem.

Supplies: a_1 =N(50,4), a_2 =N(60, 1),

a_3 = N(55, 4)

Demands: b_1 = exp(18), b_2 = exp(15),

b_3 = exp(13)

Conveyance capacities: e_1 = U(60, 80),

e_2 = U(50, 80)

Table- 1 shows the penalty matrix C^1 for the first criterion consisting of 2 sources, 2 destinations and 2 conveyances.

Table-1

$[c^1_{l121}$, $c^1_{R121}]$

	j=1		j=2	
i=1	[N(7, 2), N(15,1)]		[N(9,4), N(12,1)]	
		[N(5, 1), N(13, 4)]		[N(7, 2), N(22, 1)]
i=2	[N(10,2), N(14,3)]		[N(7,2), N(12,3)]	
		[N(9, 3), N(12, 4)]		[N(5, 1), (25, 4)]

Table-2 shows the penalty matrix C^2 for the second criterion consisting of 3 sources, 3 destinations and 2 conveyances

Table-2

$[c^2_{l122}$, $c^2_{R122}]$

	j=1		j=2	
i=1	[N(6, 2), N(14,1)],		N(4,1), N(14,4)]	
		N(7,2), N(14,3)]		N(6,1), N(20,2)]
i=2	N(9,2), N(15,4)]		N(7,3), N(11,3)]	
		N(8,2), N(12,3)]		N(6,1), N(23,2)]

Probability level: α_i = 0.9, β_j = 0.9, γ_k = 0.9, α_{Li} = 0.9, β_{Lj} = 0.9, γ_{Lk} = 0.9, α_{Ri} = 0.9, β_{Rj} = 0.9, γ_{Rk} = 0.9 and α = 0.9 β = 0.9 for i = 1, 2. j = 1, 2. and k = 1, 2.

8.1 DCCGP MODELS

8.1.1 DCGPM FOR THE FIRST CRITERION

Minimize = $\{ C^{1-}_R, C^{1-}_C \}$ subject to

$$\phi\left(\frac{Z_1}{\sqrt{Z_2}}\right) + C^{1-}_R - C^{1+}_R = 0.9$$

$$\phi\left(\frac{Z_3}{\sqrt{Z_4}}\right) + C^{1-}_C - C^{1+}_C = 0.9$$

$$x_{111} + x_{112} + x_{121} + x_{122} \leq 47.44$$

$$x_{211} + x_{212} + x_{221} + x_{222} \leq 58.72$$

$$x_{121} + x_{122} + x_{221} + x_{222} \geq 34.53$$

$$x_{111} + x_{121} + x_{211} + x_{221} \leq 62$$

$$x_{112} + x_{122} + x_{212} + x_{222} \leq 53$$

where Z_1 = 2000 –{15x_{111}+12x_{112}+13x_{121}+22x

122

$+14x_{211} +12x_{212} +12x_{221}+25x_{222}\}$

$Z_2 = \{ x^2_{111} + x^2_{112} + 4x^2_{121} + x^2_{122}$

$+3x^2_{211}+ 4x^2_{212}+ 3x^2_{221}+ 4x^2_{222} \}$

$Z_3 = 2300$--$\{11x_{111}+10.5x_{112}+9x_{121}+14.5x_{122}$

$+12x_{211} +10.5x_{212} +9.5x_{221}+15.5x_{222}\}$

$Z_4 = \{1.5 x^2_{111} +2.5 x^2_{112} +2.5x^2_{121} +1.5 x^2_{122}$

$+ 2.5x^2_{211}+ 3.5x^2_{212}+ 2.5x^2_{221}+2.5x^2_{222}\}$

with $x_{ijk} \geq 0$, for i= 1, 2. j = 1, 2. k = 1,2.

$C^{1-}_R , C^{1+}_R , C^{1-}_C , C^{1+}_C \geq 0.$

8.1.2 DCGPM FOR THE SECOND CRITERION

Minimize = $\{ C^{2-}_R ,\ C^{2-}_C \}$ subject to

$$\phi\left(\frac{Z_1}{\sqrt{Z_2}}\right) + C^{2-}_R - C^{2+}_R = 0.9$$

$$\phi\left(\frac{Z_3}{\sqrt{Z_4}}\right) + C^{2-}_C - C^{2+}_C = 0.9$$

$$x_{111} + x_{112} + x_{121} + x_{122} \leq 47.44$$

$$x_{211} + x_{212} + x_{221} + x_{222} \leq 58.72$$

$$x_{111} + x_{112} + x_{211} + x_{212} \geq 41.44$$

$$x_{121} + x_{122} + x_{221} + x_{222} \geq 34.53$$

$$x_{111} + x_{121} + x_{211} + x_{221} \leq 62$$

$$x_{112} + x_{122} + x_{212}+ x_{222} \leq 53$$

where $Z_1 = 1900 - \{14x_{111}+14x_{112}+14x_{121}+20x_{122}$

$+15x_{211} +12x_{212} +11x_{221}+23x_{222} +\}$

$Z_2 =\{ x^2_{111} +3 x^2_{112} + 4x^2_{121} + 2x^2_{122} + + 4x^2_{211}$

$+ 3x^2_{212}+ 3x^2_{221}+ 2x^2_{222}$

$Z_3 = 2400 - \{10x_{111}+10.5x_{112}+9x_{121}+13x_{122}$

$+12x_{211}+10x_{212} +9x_{221}+14.5x_{222}\}$

$Z_4 = \{1.5 x^2_{111} +2.5 x^2_{112} + 2.5x^2_{121} +1.5x^2_{122} + 3x^2_{211}$

$+2.5x^2_{212}+ 3x^2_{221}+ 1.5x^2_{222}\}$

where $x_{ijk} \geq 0$, for i= 1, 2. j = 1, 2. k = 1,2.

$C^{2-}_R ,C^{2+}_R , C^{2-}_C , C^{2+}_C \geq 0.$

9. CONCLUSION

This paper emphasizes the Dependent chance goal programming models for multi-objective interval solid transportation problem under stochastic environment in which the cost coefficients of the objective functions are in the form of stochastic intervals. Dependent chance goal programming model have been developed for two different criterions. The numerical examples have been given for each model developed in this paper. The multi-model solutions for the models can be obtained by any suitable evolutionary technique which could be the interest of the researchers.

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