Intuitionistic Fuzzy Optimization of Truss Design: A Comparative Study

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***Abstract*—*In this paper, we have developed an intuitionistic fuzzy optimization (IFO) approach considering non-linear membership and non-membership function for optimizing the design of plane truss structure with single objectives subject to a specified set of constraints. In this optimum design formulation, the objective functions are the weight of the truss; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. A classical truss optimization example is presented here in to demonstrate the efficiency of the Intuitionistic fuzzy optimization approach with non-linear membership function. We made a comparative study of linear and non-linear membership and non-membership function to see its impact on intuitionistic fuzzy optimization and to get to the depth of such optimization process. The test problem consists of a two-bar planar truss subjected to a single load condition. This single-objective structural optimization model is solved by intuitionistic fuzzy optimization approach with non-linear membership and non-membership function. Numerical example is given to illustrate our approach. The result shows that the IFO approach is very efficient in finding the best discovered optimal solutions.*

Keywords— *Intuitionistic fuzzy optimization, Non-linear membership function, Non-linear non-membership function, Structural design.*

I. INTRODUCTION

The research area of optimal structural design has been receiving increasing attention from both academia and industry over the past four decades in order to improve structural performance and to reduce design costs. However, in the real world, uncertainty or vagueness is prevalent in the Engineering Computations. In the context of structural design the uncertainty is connected with lack of accurate data of design factors. This problem has been solving by use of fuzzy mathematical algorithm for dealing with this class of problems. However the problem may be an optimization problem where one or more constraints are simultaneously satisfied subject to the minimization of the weight function. Bellman[14] and Zadeh [11] incorporate the fuzzy set theory to the decision making problem. The fuzzy set theory also found application in Structural Model. Several researchers like Wang et al. [16], Rao [13] ,Yeh et al. [18], Xu [17], Shih et.al [4], Dey et. al [5] ,Huang et.al [6] have distinctive contribution to fuzzy set theory as well as fuzzy optimization. In view of growing use of fuzzy set in optimization problem under imprecise environment, various extensions of fuzzy sets have been taken part. In such extension, Atanassov [3,8-10] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory characterized by a membership function, a non-membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. The concept of membership and non-membership was first considered by Angelov[1,2] in optimization problem and gave intuitionistic fuzzy approach to solve this. Luo.et.al [19] applied the inclusion degree of intuitionistic fuzzy set to multi criteria decision making problem. Pramanik and Roy [12] solved a vector optimization problem using an intuitionistic fuzzy goal programming. A transportation model was solved by Jana and Roy [7] using multi-objective intuitionistic fuzzy linear programming. Dey et. al [15] use Intuitionistic fuzzy optimization technique to optimize non-linear single objective two bar truss structural model.

In this paper, a well-known two bar truss design model is considered as a Structural design model. The results are compared numerically with both in fuzzy optimization technique and intuitionistic fuzzy optimization technique for non-linear membership function. From our numerical result, it is clear that intuitionistic fuzzy optimization provides better results than fuzzy optimization. The motivation of the present study is to give computational algorithm for solving single objective nonlinear programming problem by Intuitionistic fuzzy optimization approach and the impact of various type of membership functions in computation of Intuitionistic fuzzy optimization and thus made comparative study of linear and nonlinear membership.

II. SINGLE-OBJECTIVE STRUCTURAL MODEL

In sizing optimization problems, the aim is to minimize single objective function, usually the weight of the structure under certain behavioural constraints which are displacement or stresses. The design variables are most frequently chosen to be dimensions of the cross sectional areas of the members of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness of cross-sections to a certain set. A discrete structural optimization problem can be formulated in the following form

$$Minimize\ \ WT(A) \qquad (1)$$
$$subject\ to\ \sigma_i(A) \le [\sigma_i(A)], i = 1,2,\ldots\ldots,m$$
$$A_j \in R^d,\quad j = 1,2,\ldots\ldots,n$$

where $WT(A)$ represents objective function, $\sigma_i(A)$ is the behavioural constraints and $[\sigma_i(A)]$ denotes the maximum allowable value , m and n are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective function is taken as

$$WT(A) = \sum_{i=1}^{m} \rho_i l_i A_i$$

and constraint are chosen to be stress of structures as follows

$\sigma_i(A) \lesssim \sigma_i$ with allowable tolerance

σ_i^0 for $i = 1,2,\ldots.,m$

Where ρ_i and l_i are weight of unit volume and length of i^{th} element respectively, m is the number of structural element, σ_i and σ_i^0 are the i^{th} stress , allowable stress respectively.

III. Prerequisite mathematics

A. Fuzzy Set

Let X is a set (space), with a generic element of X denoted by x , that is $X(x)$ .Then a Fuzzy set (FS) is defined as $\tilde{A} = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of FS $\tilde{A}$. $\mu_{\tilde{A}}(x)$ is the degree of membership of the element x to the set $\tilde{A}$.

B. α -Level Set or α -cut of a Fuzzy Set

The α -level set of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values greater than or equal to α i.e. $\tilde{A} = \{x : \mu_{\tilde{A}}(x) \ge \alpha,\ x \in X, \alpha \in [0,1]\}$.

C. Intuitionistic Fuzzy Set

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An intuitionistic fuzzy set (IFS) set A^i in the sense of Attanassove [14] is given by equation $A^i = \{< X, \mu_{A^i}(x), \upsilon_{A^i}(x) > | x_i \in X\}$ where the function $\mu_{A^i}(x^i) : X \to [0,1]$; $x_i \in X \to \mu_{A^i}(x_i) \in [0,1]$ and $\upsilon_{A^i}(x^i) : X \to [0,1]$; $x_i \in X \to \upsilon_{A^i}(x_i) \in [0,1]$ define the degree of membership and degree of non-membership of an element $x_i \in X$ to the set $A^i \subseteq X$,such that they satisfy the condition $0 \le \mu_{A^i}(x_i) + \upsilon_{A^i}(x_i) \le 1$, $\forall\ x_i \in X$. For each IFS A^i in X the amount $\Pi_{A^i}(x_i) = 1 - (\mu_{A^i}(x^i) + \upsilon_{A^i}(x^i))$ is called the degree of uncertainty (or hesitation) associated with the membership of elements $x_i \in X$ in A^i we call it intuitionistic fuzzy index of A^i with respect of an element $x_i \in X$.

D. (α,β) – level intervals or (α,β) – cuts

A set of (α,β) – cut, generated by an IFS A^i where $(\alpha,\beta) \in [0,1]$ are fixed number such that $\alpha + \beta \le 1$ is defined as

$$A^i_{\alpha,\beta} = \left\{ \begin{array}{c} < x, \mu_{A^i}(x), \upsilon_{A^i}(x) > / x \in X \\ \mu_{A^i}(x) \ge \alpha, \upsilon_{A^i}(x) \le \beta, \alpha, \beta \in [0,1] \end{array} \right\}$$

.We define (α,β) – level or (α,β) – cut ,denoted by $A^i_{\alpha,\beta}$,as the crisp set of elements x which belong to A^i at least to the degree α and which belong to A^i at most to the degree α .

IV. MATHEMATICAL ANALYSIS

1) Intuitionistic Optimization Technique to solve Minimization Type Single Objective Non-linear Programming Problem

Let us consider a single-objective nonlinear optimization problem as

$$Minimize\ \ f(x) \qquad (2)$$
$$g_j(x) \le b_j \qquad j = 1,2,\ldots\ldots,m$$
$$x \ge 0$$

Usually constraints goals are considered as fixed quantity .But in real life problem ,the constraint goal can not be always exact. So we can consider the constraint goal for less than type constraints at least b_j and it may possible to extend to $b_j + b_j^0$ .This fact seems to take the constraint goal as a intuitionistic fuzzy set and which will be more realistic descriptions than others. Then the NLP becomes IFO problem with intuitionistic resources, which can be described as follows

$$Minimize \quad f(x) \tag{3}$$
$$g_j(x) \le \tilde{b}_j^i \qquad j = 1,2,....,m$$
$$x \ge 0$$

To solve the IFO (3), following warner's (1987) and Angelov (1995) we are presenting a solution procedure for single-objective IFO problem (3) as follows

Step-1: Following warner's approach solve the single objective non-linear programming problem without tolerance in constraints (i.e $g_j(x) \le b_j$),with tolerance of acceptance in constraints (i.e $g_j(x) \le b_j + b_j^0$) by appropriate non-linear programming technique

Here they are

Sub-problem-1

$$Minimize \quad f(x) \tag{4}$$
$$g_j(x) \le b_j \qquad j = 1,2,....,m$$
$$x \ge 0$$

Sub-problem-2

$$Minimize \quad f(x) \tag{5}$$
$$g_j(x) \le b_j + b_j^0, \quad j = 1,2,....,m$$
$$x \ge 0$$

we may get optimal solutions $x^* = x^1, f(x^*) = f(x^1)$ and $x^* = x^2, f(x^*) = f(x^2)$ for sub-problem 1 and 2 respectively.

Step-2: From the result of step 1 we now find the lower bound and upper bound of objective functions. If $U^{\mu}_{f(x)}, U^{\upsilon}_{f(x)}$ be the upper bounds of truth, indeterminacy , falsity function for the objective respectively and $L^{\mu}_{f(x)}, L^{\upsilon}_{f(x)}$ be the lower bound of membership and non-membership functions of objective respectively then

$$U^{\mu}_{f(x)} = \max\{f(x^1), f(x^2)\}, L^{\mu}_{f(x)} = \min\{f(x^1), f(x^2)\},$$
$$U^{\upsilon}_{f(x)} = U^{\mu}_{f(x)}, L^{\upsilon}_{f(x)} = L^{\mu}_{f(x)} + \varepsilon_{f(x)} \quad where \ 0 < \varepsilon_{f(x)} < \left(U^{\mu}_{f(x)} - L^{\mu}_{f(x)}\right)$$

Step-3: In this step we calculate linear membership for membership and non membership functions of objective as follows

$$\mu_{f(x)}(f(x)) = \begin{cases} 1 & if \ f(x) \le L^{\mu}_{f(x)} \\ \left(\dfrac{U^{\mu}_{f(x)} - f(x)}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)}}\right) & if \ L^{\mu}_{f(x)} \le f(x) \le U^{\mu}_{f(x)} \\ 0 & if \ f(x) \ge U^{\mu}_{f(x)} \end{cases}$$

$$\upsilon_{f(x)}(f(x)) = \begin{cases} 0 & if \ f(x) \le L^{\upsilon}_{f(x)} \\ \dfrac{f(x) - L^{\upsilon}_{f(x)}}{U^{\upsilon}_{f(x)} - L^{\upsilon}_{f(x)}} & if \ L^{\upsilon}_{f(x)} \le f(x) \le U^{\upsilon}_{f(x)} \\ 1 & if \ f(x) \ge U^{\upsilon}_{f(x)} \end{cases}$$

and exponential and hyperbolic membership for membership and non-membership functions as follows

$$\mu_{f(x)}(f(x)) = \begin{cases} 1 & if \ f(x) \le L^{\mu}_{f(x)} \\ 1 - \exp\left\{-\psi\left(\dfrac{U^{\mu}_{f(x)} - f(x)}{U^{\mu}_{f(x)} - L^{\mu}_{f(x)}}\right)\right\} & if \ L^{\mu}_{f(x)} \le f(x) \le U^{\mu}_{f(x)} \\ 0 & if \ f(x) \ge U^{\mu}_{f(x)} \end{cases}$$

$$\upsilon_{f(x)}(f(x)) = \begin{cases} 0 & if \ f(x) \le L^{\upsilon}_{f(x)} \\ \dfrac{1}{2} + \dfrac{1}{2}\tanh\left\{\left(f(x) - \dfrac{U^{\upsilon}_{f(x)} + L^{\upsilon}_{f(x)}}{2}\right)\tau_{f(x)}\right\} & if \ L^{\upsilon}_{f(x)} \le f(x) \le U^{\upsilon}_{f(x)} \\ 1 & if \ f(x) \ge U^{\upsilon}_{f(x)} \end{cases}$$

Step-4: In this step using linear, exponential and hyperbolic function for membership and non-membership functions, we may calculate membership function for constraints as follows

$$\mu_{g_j(x)}(g_j(x)) = \begin{cases} 1 & if \ g_j(x) \le b_j \\ \left(\dfrac{b_j + b_j^0 - g_j(x)}{b_j^0}\right) & if \ b_j \le g_j(x) \le b_j + b_j^0 \\ 0 & if \ g_j(x) \ge b_j^0 \end{cases}$$

$$\upsilon_{g_j(x)}(g_j(x)) = \begin{cases} 0 & if \ g_j(x) \le b_j + \varepsilon_{g_j(x)} \\ \dfrac{g_j(x) - b_j - \varepsilon_{g_j(x)}}{b_j^0 - \varepsilon_{g_j(x)}} & if \ b_j + \varepsilon_{g_j(x)} \le g_j(x) \le b_j + b_j^0 \\ 1 & if \ g_j(x) \ge b_j + b_j^0 \end{cases}$$

where and for $j = 1,2,.....,m \quad 0 < \varepsilon_{g_j(x)}, \xi_{g_j(x)} < b_j^0$.

and

$$\mu_{g_j(x)}(g_j(x)) = \begin{cases} 1 & if \ g_j(x) \le b_j \\ 1 - \exp\left\{-\psi\left(\dfrac{U^{\mu}_{g_j(x)} - g_j(x)}{U^{\mu}_{g_j(x)} - L^{\mu}_{g_j(x)}}\right)\right\} & if \ b_j \le g_j(x) \le b_j + b_j^0 \\ 0 & if \ g_j(x) \ge b_j + b_j^0 \end{cases}$$

$$\upsilon_{g_j(x)}\left(g_j(x)\right)=\begin{cases}0 & if\ g_j(x)\le b_j+\varepsilon_{g_j(x)}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(g_j(x)-\frac{2b_j+b_j^0+\varepsilon_{g_j(x)}}{2}\right)\tau_{g_j(x)}\right\} & if\ b_j+\varepsilon_{g_j(x)}\le g_j(x)\le b_j+b_j^0\\ 1 & if\ g_j(x)\ge b_j+b_j^0\end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for $j=1,2,.....,m$ $0<\varepsilon_{g_j(x)},\xi_{g_j(x)}<b_j^0$.

Step-5: Now using IFO for single objective optimization technique the optimization problem (2) can be formulated as

Model-I

$$Maximize\ (\alpha-\beta) \tag{6}$$

Suh that

$$\mu_{f(x)}(x)\ge\alpha;\ \mu_{g_j}(x)\ge\alpha;$$
$$\upsilon_{f(x)}(x)\le\beta;\ \upsilon_{g_j}(x)\le\beta;$$
$$\alpha+\beta\le 1;\ \alpha\ge\beta$$
$$\alpha,\beta\in[0,1]$$

Where

$$\alpha=\mu_{\tilde{D}^i}(x)=\min\left\{\mu_{f(x)}\left(f(x)\right),\mu_{g_j(x)}\left(g_j(x)\right)\right\}\ for\ j=1,2,.......,m$$

and

$$\beta=\upsilon_{\tilde{D}^n}(x)=\max\left\{\upsilon_{f(x)}\left(f(x)\right),\upsilon_{g_j(x)}\left(g_j(x)\right)\right\}\ for\ j=1,2,.......,m$$

are the membership and nonmembership function of decision set $\tilde{D}^i=f^i(x)\bigcap_{j=1}^{m}g^i{}_j(x)$

Now the above problem (6) can be simplified to following crisp linear programming problem for linear membership function as

$$Maximize\ (\alpha-\beta) \tag{7}$$

$$such\ that\ f(x)+\left(U^\mu-L^\mu\right).\alpha\le U^\mu;$$
$$f(x)+\left(U^\upsilon_{f(x)}-L^\upsilon_{f(x)}\right).\beta\le L^\upsilon_{f(x)};$$
$$\alpha+\beta\le 1;\alpha\ge\beta;\alpha,\beta\in[0,1];$$
$$g_j(x)\le b_j\quad x\ge 0,$$

and for non linear membership function as

$$Maximize\ (\theta-\eta) \tag{8}$$

Such that

$$f(x)+\theta\frac{\left(U^\mu_{f(x)}-L^\mu_{f(x)}\right)}{\psi}\le U^\mu_{f(x)};$$
$$f(x)+\frac{\eta}{\tau_{f(x)}}\le\frac{U^\mu_{f(x)}+L^\mu_{f(x)}+\varepsilon_{f(x)}}{2};$$
$$g_j(x)+\theta\frac{b_j^0}{\psi}\le b_j+b_j^0;$$
$$g_j(x)+\frac{\eta}{\tau_{g(x)}}\le\frac{2b_j+b_j^0+\varepsilon_{g_j(x)}}{2};$$
$$\theta+\eta\le 1;$$
$$\theta\ge\eta;$$
$$\theta,\eta\in[0,1]$$

where $\theta=-\ln(1-\alpha)$; $\psi=4$; $\tau_{f(x)}=\frac{6}{\left(U^\upsilon_{f(x)}-L^\upsilon_{f(x)}\right)}$;

$\tau_{g_j(x)}=\frac{6}{\left(b_j^0-\varepsilon_j\right)}$, $for\ j=1,2,........,m$

$\eta=-\tanh^{-1}(2\beta-1)$. for linear and nonlinear membership function respectively .

All these crisp nonlinear programming problems (7),(8) can be solved by appropriate mathematical algorithm..

*2) **Solution of Single Objective Structural Optimization Problem (SOSOP)by Intuitionistic Fuzzy Optimization Technique***

The To solve the SOSOP (1), step 1 of 1 is used and we will get optimum solutions of two sub problem as A^1 and A^2 .After that according to step 2 we find upper and lower bound of membership function of objective function as $U^\mu_{WT(A)},U^\upsilon_{WT(A)}$ and $L^\mu_{WT(A)},L^\upsilon_{WT(A)}$

where $U^\mu_{WT(A)}=\max\left\{WT\left(A^1\right),WT\left(A^2\right)\right\}$, $L^\mu_{WT(A)}=\min\left\{WT\left(A^1\right),WT\left(A^2\right)\right\}$, $U^\upsilon_{WT(A)}=U^\mu_{WT(A)},L^\upsilon_{WT(A)}=L^\mu_{WT(A)}+\varepsilon_{WT(A)}$ where $0<\varepsilon_{WT(A)}<\left(U^\mu_{WT(A)}-L^\mu_{WT(A)}\right)$

Let the linear membership function for objective be

$$\mu_{WT(A)}\left(WT(A)\right)=\begin{cases}1 & if\ WT(A)\le L^\mu_{WT(A)}\\ \left(\frac{U^\mu_{WT(A)}-WT(A)}{U^\mu_{WT(A)}-L^\mu_{WT(A)}}\right) & if\ L^\mu_{WT(A)}\le WT(A)\le U^\mu_{WT(A)}\\ 0 & if\ WT(A)\ge U^\mu_{WT(A)}\end{cases}$$

$$\upsilon_{WT(A)}\left(WT(A)\right)=\begin{cases}0 & if\ WT(A)\le L^\mu_{WT(A)}+\varepsilon_{WT(A)}\\ \left(\frac{WT(A)-\left(L^\mu_{WT(A)}+\varepsilon_{WT(A)}\right)}{U^\mu_{WT(A)}-L^\mu_{WT(A)}-\varepsilon_{WT(A)}}\right) & if\ L^\mu_{WT(A)}+\varepsilon_{WT(A)}\le WT(A)\le U^\mu_{WT(A)}\\ 1 & if\ WT(A)\ge U^\mu_{WT(A)}\end{cases}$$

and constraints be

$$\mu_{\sigma_i(A)}\left(\sigma_i(A)\right)=\begin{cases}1 & if\ \sigma_i(A)\le\sigma_i\\ \left(\frac{\left(\sigma_i+\sigma_i^0\right)-\sigma_i(A)}{\sigma_i^0}\right) & if\ \sigma_i\le\sigma_i(A)\le\sigma_i+\sigma_i^0\\ 0 & if\ \sigma_i(A)\ge\sigma_i+\sigma_i^0\end{cases}$$

$$\upsilon_{\sigma_i(A)}\left(\sigma_i\left(A\right)\right)=\begin{cases} 0 & if\ \ \sigma_i\left(A\right)\le\sigma_i+\varepsilon_{\sigma_i(x)} \\ \left(\dfrac{\sigma_i\left(A\right)-\sigma_i-\varepsilon_{\sigma_i(x)}}{\sigma_i^0-\varepsilon_{\sigma_i(x)}}\right) & if\ \ \sigma_i+\varepsilon_{\sigma_i(x)}\le\sigma_i\left(A\right)\le\sigma_i+\sigma_i^0 \\ 1 & if\ \ \sigma_i\left(A\right)\ge\sigma_i+\sigma_i^0 \end{cases}$$

where for $j=1,2,...,m\ \ 0<\varepsilon_{\sigma_i(x)},\xi_{\sigma_i(A)}<\sigma_i^0$
and if non-linear membership function be considered for objective function $WT\left(A\right)$ then

$$\mu_{WT(A)}\left(WT\left(A\right)\right)=$$

$$\begin{cases} 1 & if\ \ WT\left(A\right)\le L^{\mu}_{WT(A)} \\ 1-\exp\left\{-\psi\left(\dfrac{U^{\mu}_{WT(A)}-WT\left(A\right)}{U^{\mu}_{WT(A)}-L^{\mu}_{WT(A)}}\right)\right\} & if\ \ L^{\mu}_{WT(A)}\le WT\left(A\right)\le U^{\mu}_{WT(A)} \\ 0 & if\ \ WT\left(A\right)\ge U^{\mu}_{WT(A)} \end{cases}$$

$$\upsilon_{WT(A)}\left(WT\left(A\right)\right)=$$

$$\begin{cases} 0 & if\ WT\left(A\right)\le L^{\mu}_{WT(A)}+\varepsilon_{WT} \\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(WT\left(A\right)-\dfrac{\left(U^{\mu}_{WT(A)}+L^{\mu}_{WT(A)}\right)+\varepsilon_{WT}}{2}\right)\tau_{WT}\right\} & if\ \ L^{\mu}_{WT(A)}+\varepsilon_{WT}\le WT\left(A\right)\le U^{\mu}_{WT(A)} \\ 1 & if\ \ WT\left(A\right)\ge U^{\mu}_{WT(A)} \end{cases}$$

where $0<\varepsilon_{WT},\xi_{WT}<\left(U^{\mu}_{WT}-L^{\mu}_{WT}\right)$ and if nonlinear truth, indeterminacy and falsity membership functions be considered for constraints then

$$\mu_{\sigma_i(A)}\left(\sigma_i\left(A\right)\right)=\begin{cases} 1 & if\ \ \sigma_i\left(A\right)\le L^{\mu}_{\sigma_i} \\ 1-\exp\left\{-\psi\left(\dfrac{U^{\mu}_{\sigma_i}-\delta\left(A\right)}{U^{\mu}_{\sigma_i}-L^{\mu}_{\sigma_i}}\right)\right\} & if\ \ L^{\mu}_{\sigma_i}\le\sigma_i\left(A\right)\le U^{\mu}_{\sigma_i} \\ 0 & if\ \ \sigma_i\left(A\right)\ge U^{\mu}_{\sigma_i} \end{cases}$$

$$\upsilon_{\sigma_i(A)}\left(\sigma_i\left(A\right)\right)=$$

$$\begin{cases} 0 & if\ \ \sigma_i\left(A\right)\le L^{\mu}_{\sigma_i}+\varepsilon_{\sigma_i} \\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\sigma_i\left(A\right)-\dfrac{\left(U^{\mu}_{\sigma_i}+L^{\mu}_{\sigma_i}\right)+\varepsilon_{\sigma_i}}{2}\right)\tau_{\sigma_i}\right\} & if\ \ L^{\mu}_{\sigma_i}+\varepsilon_{\sigma_i}\le\sigma_i\left(A\right)\le U^{\mu}_{\sigma_i} \\ 1 & if\ \ \sigma_i\left(A\right)\ge U^{\mu}_{\sigma_i} \end{cases}$$

for objectives where ψ,τ are non-zero parameters prescribed by the decision maker and for *where* $0<\varepsilon_{\sigma_i},\xi_{\sigma_i}<\left(U^{\mu}_{\sigma_i}-L^{\mu}_{\sigma_i}\right)$ then intuitionistic optimization problem can be formulated as

Model-I

Maximize $\left(\alpha-\beta\right)$

such that

$\mu_{WT(A)}\left(WT\left(A\right)\right)\ge\alpha;\ \ \mu_{\sigma_i(A)}\left(\sigma_i\left(A\right)\right)\ge\alpha;$

$\upsilon_{WT(A)}\left(WT\left(A\right)\right)\le\beta;\ \upsilon_{\sigma_i(A)}\left(\sigma_i\left(A\right)\right)\le\beta$

$\sigma_i\left(x\right)\le\left[\sigma_i\right];\ \alpha+\beta\le1;\ \alpha\ge\beta;\ \alpha,\beta\in\left[0,1\right]$

And now the above problem can be simplified to following crisp linear programming problem, whenever linear membership are considered, as

Model-IA

Maximize $\left(\alpha-\beta\right)$ (9)

Such that

$WT\left(A\right)+\alpha\left(U^{\mu}_{WT(A)}-L^{\mu}_{WT(A)}\right)\le U^{\mu}_{WT(A)};$

$WT\left(A\right)-\beta\left(U^{\mu}_{WT(A)}-L^{\mu}_{WT(A)}-\varepsilon_{WT(A)}\right)\le L^{\mu}_{WT(A)}+\varepsilon_{WT(A)};$

$\sigma_T\left(A\right)+\alpha\left(U^{\mu}_{\sigma_T(A)}-L^{\mu}_{\sigma_T(A)}\right)\le U^{\mu}_{\sigma_T(A)};$

$\sigma_T\left(A\right)-\beta\left(U^{\mu}_{\sigma_T(A)}-L^{\mu}_{\sigma_T(A)}-\varepsilon_{\sigma_T(A)}\right)\le L^{\mu}_{\sigma_T(A)}+\varepsilon_{\sigma_T(A)};$

$\sigma_C\left(A\right)+\alpha\left(U^{\mu}_{\sigma_C(A)}-L^{\mu}_{\sigma_C(A)}\right)\le U^{\mu}_{\sigma_C(A)};$

$\sigma_C\left(A\right)-\beta\left(U^{\mu}_{\sigma_C(A)}-L^{\mu}_{\sigma_C(A)}-\varepsilon_{\sigma_C(A)}\right)\le L^{\mu}_{\sigma_C(A)}+\varepsilon_{\sigma_C(A)};$

$\alpha+\beta\le1;\ \alpha\ge\beta;\ \alpha,\beta\in\left[0,1\right]$

And crisp linear programming problem whenever non-linear membership function is considered as

Model-IB

Maximize $\left(\theta-\eta\right)$ (10)

such that

$$WT\left(A\right)+\theta\frac{\left(U^{\mu}_{WT(A)}-L^{\mu}_{WT(A)}\right)}{\psi}\le U^{\mu}_{WT(A)};$$

$$WT\left(A\right)+\frac{\eta}{\tau_{WT(A)}}\le\frac{U^{\mu}_{WT(A)}+L^{\mu}_{WT(A)}+\varepsilon_{WT(A)}}{2};$$

$$\sigma_i\left(A\right)+\theta\frac{\sigma_i^0}{\psi}\le\sigma_i+\sigma_i^0;$$

$$\sigma_i\left(A\right)+\frac{\eta}{\tau_{\sigma_i(A)}}\le\frac{2\sigma_i+\sigma_i^0+\varepsilon_{\sigma_i(A)}}{2};$$

$\theta+\eta\le1;\ \theta\ge\eta;\ \theta,\eta\in\left[0,1\right]$

Where

$$\theta=-\ln\left(1-\alpha\right);\quad\psi=4;\quad\tau_{WT(A)}=\frac{6}{\left(U^{\upsilon}_{WT(A)}-L^{\upsilon}_{WT(A)}\right)};$$

$$\eta=-\tanh^{-1}\left(2\beta-1\right).\ and\ \tau_{\sigma_i(A)}=\frac{6}{\left(U^{\upsilon}_{\sigma_i(A)}-L^{\upsilon}_{\sigma_i(A)}\right)};.$$

All these crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

V. NUMERICAL ILLUSTRATION

A well-known two-bar planar truss structure (Fig.1.)is considered. The design objective is to minimize weight of the structural $WT\left(A_1,A_2,y_B\right)$ of a statistically loaded two-bar truss subjected to stress $\sigma_i(A_1, A_2, y_B)$ constraints on each of the truss members $i = 1, 2$.

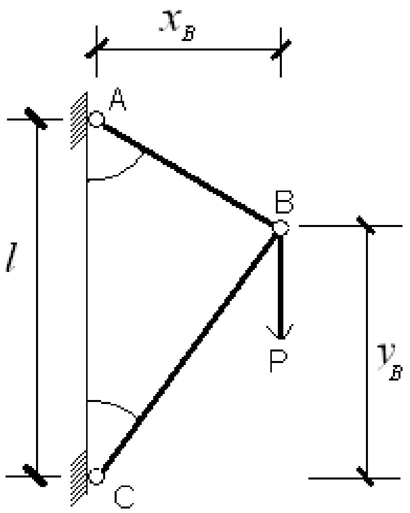

Fig.1. Design of the two-bar truss

The multi-objective optimization problem can be stated as follows

$$\text{Minimize } WT(A_1, A_2, y_B) = \rho\left(A_1\sqrt{x_B^2 + (l - y_B)^2} + A_2\sqrt{x_B^2 + y_B^2}\right) \quad (11)$$

Such that

$$\sigma_{AB}(A_1, A_2, y_B) \equiv \frac{P\sqrt{x_B^2 + (l - y_B)^2}}{lA_1} \leq \left[\sigma_{AB}^T\right];$$

$$\sigma_{BC}(A_1, A_2, y_B) \equiv \frac{P\sqrt{x_B^2 + y_B^2}}{lA_2} \leq \left[\sigma_{BC}^C\right];$$

$$0.5 \leq y_B \leq 1.5$$

$$A_1 > 0, A_2 > 0;$$

where $P =$ nodal load ; $\rho =$ volume density ; $l =$ length of AC ; $x_B =$ perpendicular distance from AC to point B. $A_1 =$ Cross section of bar- AB ; $A_2 =$ Cross section of bar- BC . $[\sigma_T] =$ maximum allowable tensile stress , $[\sigma_C] =$ maximum allowable compressive stress and $y_B = y$ -co-ordinate of node B .Input data are given in table 1.

Solution : According to step 2 of 1,we find upper and lower bound of membership function of objective function as $U_{WT(A)}^{\mu}, U_{WT(A)}^{\upsilon}$ and $U_{WT(A)}^{\mu}, U_{WT(A)}^{\upsilon}$

where $U_{WT(A)}^{\mu} = 14.23932 = U_{WT(A)}^{\upsilon}, L_{WT(A)}^{\mu} = 12.57667$

$L_{WT(A)}^{\upsilon} = 12.57667 + \varepsilon_{WT(A)}$ where $0 < \varepsilon_{WT(A)} < 1.66265;$

Now using the bounds we calculate the membership functions for objective as follows

$$\mu_{WT(A_1,A_2,y_B)}\left(WT(A_1,A_2,y_B)\right) = \begin{cases} 1 & \text{if } WT(A_1,A_2,y_B) \leq 12.57667 \\ \left(\dfrac{14.23932 - WT(A_1,A_2,y_B)}{1.66265}\right) & \text{if } 12.57667 \leq WT(A_1,A_2,y_B) \leq 14.23932 \\ 0 & \text{if } WT(A_1,A_2,y_B) \geq 14.23932 \end{cases}$$

$$\upsilon_{WT(A_1,A_2,y_B)}\left(WT(A_1,A_2,y_B)\right) = \begin{cases} 0 & \text{if } WT(A_1,A_2,y_B) \leq 12.57667 + \varepsilon_{WT(A)} \\ \left(\dfrac{WT(A_1,A_2,y_B) - 12.57667 - \varepsilon_{WT(A)}}{1.66265 - \varepsilon_{WT(A)}}\right) & \text{if } 12.57667 + \varepsilon_{WT(A)} \leq WT(A_1,A_2,y_B) \leq 14.23932 \\ 1 & \text{if } WT(A_1,A_2,y_B) \geq 14.23932 \end{cases}$$

Similarly the membership functions for tensile stress are

$$\mu_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right) = \begin{cases} 1 & \text{if } \sigma_T(A_1,A_2,y_B) \leq 130 \\ \left(\dfrac{150 - \sigma_T(A_1,A_2,y_B)}{20}\right) & \text{if } 130 \leq \sigma_T(A_1,A_2,y_B) \leq 150 \\ 0 & \text{if } \sigma_T(A_1,A_2,y_B) \geq 150 \end{cases}$$

$$\upsilon_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right) = \begin{cases} 0 & \text{if } \sigma_T(A_1,A_2,y_B) \leq 130 + \varepsilon_{\sigma_T} \\ \left(\dfrac{\sigma_T(A_1,A_2,y_B) - 130 - \varepsilon_{\sigma_T}}{20 - \varepsilon_{\sigma_T}}\right) & \text{if } 130 + \varepsilon_{\sigma_T} \leq \sigma_T(A_1,A_2,y_B) \leq 150 \\ 1 & \text{if } \sigma_T(A_1,A_2,y_B) \geq 150 \end{cases}$$

where $0 < \varepsilon_{\sigma_T}, \xi_{\sigma_T} < 20$

and the membership functions for compressive stress constraint are

$$\mu_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right) = \begin{cases} 1 & \text{if } \sigma_C(A_1,A_2,y_B) \leq 90 \\ \left(\dfrac{100 - \sigma_C(A_1,A_2,y_B)}{10}\right) & \text{if } 90 \leq \sigma_C(A_1,A_2,y_B) \leq 100 \\ 0 & \text{if } \sigma_C(A_1,A_2,y_B) \geq 100 \end{cases}$$

$$\upsilon_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right) = \begin{cases} 0 & \text{if } \sigma_C(A_1,A_2,y_B) \leq 90 + \varepsilon_{\sigma_C} \\ \left(\dfrac{\sigma_C(A_1,A_2,y_B) - 90 - \varepsilon_{\sigma_C}}{10 - \varepsilon_{\sigma_C}}\right) & \text{if } 90 + \varepsilon_{\sigma_C} \leq \sigma_C(A_1,A_2,y_B) \leq 100 \\ 1 & \text{if } \sigma_C(A_1,A_2,y_B) \geq 100 \end{cases}$$

where $0 < \varepsilon_{\sigma_C}, \xi_{\sigma_C} < 10$

Again the nonlinear membership and nonmembership functions for objectives and constraints can be formulated as

$$\mu_{WT(A_1,A_2,y_B)}\left(WT(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ WT(A_1,A_2,y_B)\le 12.57667\\ 1-\exp\left\{-4\left(\dfrac{14.23932-WT(A_1,A_2,y_B)}{1.66265}\right)\right\} & if\ 12.57667\le WT(A_1,A_2,y_B)\le 14.23932\\ 0 & if\ WT(A_1,A_2,y_B)\ge 14.23932\end{cases}$$

$$\upsilon_{WT(A_1,A_2,y_B)}\left(WT(A_1,A_2,y_B)\right)=\begin{cases}0 & if\ WT(A_1,A_2,y_B)\le 12.57667+\varepsilon_{WT}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(WT(A_1,A_2,y_B)-\dfrac{(26.81599+\varepsilon_{WT})}{2}\right)\dfrac{6}{1.66265-\varepsilon_{WT}}\right\} & if\ 12.57667+\varepsilon_{WT}\le WT(A_1,A_2,y_B)\le 14.23932\\ 1 & if\ WT(A_1,A_2,y_B)\ge 14.23932\end{cases}$$

Similarly the membership functions for tensile stress are

$$\mu_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ \sigma_T(A_1,A_2,y_B)\le 130\\ 1-\exp\left\{-4\left(\dfrac{150-\sigma_T(A_1,A_2,y_B)}{20}\right)\right\} & if\ 130\le \sigma_T(A_1,A_2,y_B)\le 150\\ 0 & if\ \sigma_T(A_1,A_2,y_B)\ge 150\end{cases}$$

$$\upsilon_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right)=\begin{cases}0 & if\ \sigma_T(A_1,A_2,y_B)\le 130+\varepsilon_{\sigma_T}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\sigma_T(A_1,A_2,y_B)-\left(\dfrac{280+\varepsilon_{\sigma_T}}{2}\right)\right)\dfrac{6}{20-\varepsilon_{\sigma_T}}\right\} & if\ 130+\varepsilon_{\sigma_T}\le \sigma_T(A_1,A_2,y_B)\le 150\\ 1 & if\ \sigma_T(A_1,A_2,y_B)\ge 150\end{cases}$$

$where\ 0<\varepsilon_{\sigma_T},\xi_{\sigma_T}<20$

and the membership and non-membership functions for compressive stress constraint are

$$\mu_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ \sigma_C(A_1,A_2,y_B)\le 90\\ 1-\exp\left\{-4\left(\dfrac{100-\sigma_C(A_1,A_2,y_B)}{10}\right)\right\} & if\ 90\le \sigma_C(A_1,A_2,y_B)\le 100\\ 0 & if\ \sigma_C(A_1,A_2,y_B)\ge 100\end{cases}$$

$$\upsilon_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right)=\begin{cases}0 & if\ \sigma_C(A_1,A_2,y_B)\le 90+\varepsilon_{\sigma_C}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\sigma_C(A_1,A_2,y_B)-\left(\dfrac{190+\varepsilon_{\sigma_C}}{2}\right)\right)\dfrac{6}{10-\varepsilon_{\sigma_C}}\right\} & if\ 90+\varepsilon_{\sigma_C}\le \sigma_C(A_1,A_2,y_B)\le 100\\ 1 & if\ \sigma_C(A_1,A_2,y_B)\ge 100\end{cases}$$

$where\ 0<\varepsilon_{\sigma_C},\xi_{\sigma_C}<10$

Now , using above mentioned truth, indeterminacy and falsity linear and nonlinear membership function NLP (11) can be solved for Model-IA, Model-IB, by IFSO technique for different values of $\varepsilon_{WT},\varepsilon_{\sigma_T},\varepsilon_{\sigma_C}$. The optimum solution of SOSOP(11) is given in table 1 and table 2 and the solution is compared with fuzzy and intuitionistic fuzzy problem.

VI. TABLE 1

INPUT DATA OF CRISP MODEL (11).

Applied load P (KN)	Volume density ρ (KN/m^3)	Length l (m)	Maximum allowable tensile stress $[\sigma_T]$ (Mpa)	Maximum allowable compressive stress $[\sigma_C]$ (Mpa)	Distance of x_B from AC (m)
100	7.7	2	130 with fuzzy region 20	90 with fuzzy region 10	1

TABLE 2

INPUT DATA OF CRISP MODEL (11).

Methods	Model	A_1 (m^2)	A_2 (m^2)	$WT(A_1, A_2)$ (KN)	y_B (m)
Fuzzy single-objective non-linear programming (FSONLP)	IA	.5883491	.7183381	14.23932	1.013955
	IB	.5883491	.7183381	14.23932	1.013955
Intuitionistic Fuzzy single-objective non-linear programming (FSONLP)	IA $\varepsilon_{WT} = 0.33253,\ \varepsilon_{\sigma_T} = 4,\ \varepsilon_{\sigma_C} = 2$	0.5482919	0.6692795	13.19429	0.8067448
	IB $\varepsilon_{WT} = 0.8,\ \varepsilon_{\sigma_T} = 16,\ \varepsilon_{\sigma_C} = 8$	0.6064095	0.6053373	13.59182	0.5211994

Here we get best solution for different tolerance $\varepsilon_{WT}, \varepsilon_{\sigma_T}$ *and* ε_{σ_C} for non linear membership and non-membership function of IFO method. From Table-2 it shows that IFO for non-linear membership gives better result in perspective of structural design.

VII. CONCLUSIONS

In view of comparing the intuitionistic fuzzy optimization with fuzzy optimization method for membership and non-membership we also obtained the solution of the undertaken numerical problem by fuzzy optimization method given by Zimmermann and intuitionistic fuzzy optimization method given by Angelov. The main objective of this work is to illustrate the impact of nonlinear membership and non-membership of IFO technique in utilization of nonlinear structural problem . Here we have considered a non-linear two bar truss design problem .In this problem, we find out optimum weight of the structure in presence of optimum deflection of loaded joint. The comparison of results obtained for the undertaken problem clearly show the difference between the linear and non-linear intuitionistic fuzzy optimization in perspective of structural design. The results of this study may lead to the development of effective non linear IFO i.e (NLIFO) technique solving other nonlinear model in different field.

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