A Study on Intuitionistic Anti L-Fuzzy Normal M-Subgroups

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Abstract–*In this paper, we introduce the concept of intuitionistic anti L-fuzzy normal M- subgroups and investigate some related properties.*

Keywords**:** *Intuitionistic fuzzy subsets; Intuitionistic anti fuzzy subgroups; Intuitionistic anti L-fuzzy M-subgroups; Intuitionistic anti L-fuzzy normal M-subgroups ;M- homomorphism.*

I. INTRODUCTION

A fuzzy set theory has developed in many directions andfinding application in a wide variety of fields. Zadeh'sclassicalpaper [20] of 1965 introduced the concepts of fuzzysets and fuzzy set operations. The study of fuzzy groups wasstarted by Rosenfeld [16] and it was extended by Roventa [17]who have introduced the concept of fuzzy groups operatingon fuzzy sets and many researchers [1,7,9,10] are engaged inextending the concepts. The concept of intuitionistic fuzzy setwas introduced by Atanassov. K.T [2,3], as a generalizationof the notion of fuzzy sets. Choudhury. F.P et al [6] defined afuzzy subgroup and fuzzy homomorphism. **Palaniappan. N** and **Muthuraj,** [11] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. **Pandiammal. P, Natarajan. R** and**Palaniappan. N,** [13] defined the homomorphism, anti-homorphism of an anti L-fuzzy M-subgroup.**Pandiammal. P,**[14] defined the homomorphism, anti-homomorphism of an intuitionistic anti L-fuzzy M-subgroups. In this paper weintroduce and discuss the algebraic nature of intuitionisticanti L-fuzzy normal M-subgroups with operator and obtain some relatedresults.

II. PRELIMINARIES

2.1 Definition: Let G be a M-group. AL-fuzzy subset A of G is said to be **anti L-fuzzyM-subgroup** (ALFMSG) of G if its satisfies the following axioms:

(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(x^{-1}) \leq \mu_A(x)$, for all x and y in G.

2.2 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an **intuitionistic L-fuzzy M-subgroup (ILFMSG)** of G if the following conditions are satisfied: (i) $\mu_A(mxy) \geq \mu_A(x) \wedge \mu_A(y)$, (ii)$\mu_A(x^{-1}) \geq \mu_A(x)$,(iii) $\nu_A(mxy) \leq \nu_A(x) \vee \nu_A(y)$,

(iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x & y in G.

2.3Definition: Let (G, ·) and (G^l, ·) be any two M-groups. Let $f : G \to G^l$ be any function and A be an intuitionistic L-fuzzy M-subgroup in G, V be an intuitionistic L-fuzzy M-subgroup in f (G) = G^l, defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$, for all x in G and y in G^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2.4 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB = { ⟨ (x, y), $\mu_{AxB}(x, y)$, $\nu_{AxB}(x, y)$ ⟩ / for all x in G and y in H }, where$\mu_{AxB}(x, y) = \mu_A(x) \wedge \mu_B(y)$ and $\nu_{AxB}(x, y) = \nu_A(x) \vee \nu_B(y)$.

2.5Definition: Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **conjugate intuitionistic L-fuzzyM-subgroups** of G if for some g in G,$\mu_A(x) = \mu_B(g^{-1}xg)$ and $\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

2.6 Definition: Let A be an intuitionistic L-fuzzy subset in a set S, the **strongest intuitionistic L-fuzzy relation** on S, that is an intuitionistic L-fuzzy relation on A is V given by $\mu_V(x, y) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_V(x, y) = \nu_A(x) \vee \nu_A(y)$, for all x and y in S.

III. INTUITIONISTIC ANTI L-FUZZY NORMAL M-SUBGROUPS

3.1Defintion: An intuitionistic fuzzy subset μ in a groupG is said to be an intuitionistic anti fuzzy subgroup of G ifthe following axioms are satisfied.

(i) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$,

(ii)$\mu_A(x^{-1}) \leq \mu_A(x)$,

(iii) $\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$,

(iv) $\nu_A(x^{-1}) \geq \nu_A(x)$, for all x and y in G.

3.2 Proposition: Let G be a group. An intuitionistic fuzzysubset μ in a group G is said to be an intuitionistic anti fuzzysubgroup of G if the following conditions are satisfied.(i)$\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, (ii) $\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$, for all x ,y inG.

3.3 Definition: Let G be an M-group and μ be anintuitionistic anti fuzzy group of G. If$\mu_A(mx) \leq \mu_A(x)$ and $\nu_A(mx) \geq \nu_A(x)$ for all x inG and m in M then μ is said to be an intuitionistic anti fuzzy subgroupwith operator of G. We use the phrase μ is an intuitionisticanti L-fuzzy M-subgroup of G.

3.4 Example: Let H be M-subgroup of an M-group Gand let A= (μ_A,ν_A)be an intuitionistic fuzzy set in Gdefined by

$$\mu_A(x) = \begin{cases} 0.3 ; & x \in H \\ 0.5; & \text{otherwise} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.6; & x \in H \\ 0.3; & \text{otherwise} \end{cases}$$

For all x in G. Then it is easy to verify that A= $(\mu_A,\nu_A)$is an anti fuzzy M- subgroup of G.

3.5 Definition: Let A and B be any two intuitionistic anti L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **conjugate intuitionistic antiL-fuzzy normal M-subgroups** of G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$ and $\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

3.6 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB = { ⟨ (x, y), $\mu_{AxB}(x, y)$, $\nu_{AxB}(x, y)$ ⟩ / for all x in G and y in H }, where $\mu_{AxB}(x, y) = \mu_A(x) \vee \mu_B(y)$ and $\nu_{AxB}(x, y) = \nu_A(x) \wedge \nu_B(y)$.

3.7Definition: Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group(G, ·). Then A and B are said to be conjugate intuitionistic L-fuzzy M-subgroups of G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$ &$\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

3.8 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy M-subgroup A of G is said to be an **intuitionistic Anti L-fuzzy normal M-subgroup (IALFNMSG)** of G if the following conditions are satisfied:

(i) $\mu_A(xy) = \mu_A(yx)$,

(ii) $\nu_A(xy) = \nu_A(yx)$, for all x and y in G.

3.9 Definition: An intuitionistic L-fuzzy subset A of a set X is said to be **normalized** if there exist x in X such that $\mu_A(x) =1$ and $\nu_A(x) =0$.

3.10Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy M-subgroup A of G is said to be an **intuitionistic L-fuzzy characteristic M-subgroup (ILFCMSG)** of G if the following conditions are satisfied:

(i) $\mu_A(x) = \mu_A(f(x))$,

(ii) $\nu_A(x) = \nu_A(f(x))$, for all x in G and f in AutG.

3.SOME PROPERTIES OF INTUITIONISTIC L-FUZZY NORMAL M-SUBGROUPS

3.1 Theorem : If A and B are two intuitionisticanti L-fuzzy M-subgroups of M-groups G, then their intersection A∩B is an intuitionistic antiL-fuzzy M-subgroup of a M-group G.

3.2 Theorem: The intersection of a family of intuitionistic anti L-fuzzy M-subgroups of M-group G is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

3.3 Theorem : If A and B are intuitionistic antiL-fuzzy M-subgroups of the M-groups G and H, respectively, then AxB is an intuitionistic antiL-fuzzy M-subgroup of GxH.

3.4 Theorem: Let an intuitionistic anti L-fuzzy M-subgroup A of a M-group G be conjugate to an intuitionistic anti L-fuzzy M-subgroup M of G and an intuitionistic anti L-fuzzy M-subgroup B of a M-group H be conjugate to an intuitionistic antiL-fuzzy M-subgroup N of H. Then an intuitionistic anti L-fuzzy M-subgroup AxB of a M-group GxH is conjugate to an intuitionistic anti L-fuzzy M-subgroup MxN of GxH.

3.5 Theorem: Let A and B be intuitionistic anti L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e^{l} are the identity elements of G and H, respectively. If AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH, then at least one of the following two statements must hold.

(i) $\mu_B(e^{l}) \leq \mu_A(x)$ and $\nu_B(e^{l}) \geq \nu_A(x)$, for all x in G,

(ii) $\mu_A(e) \leq \mu_B(y)$ and $\nu_A(e) \geq \nu_B(y)$, for all y in H.

3.6 Theorem: Let A and B be intuitionistic anti L-fuzzy subsets of the M-groups G and H, respectively and AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH. Then the following are true :

(i) if $\mu_A(x) \leq \mu_B(e^{l})$ and $\nu_A(x) \geq \nu_B(e^{l})$, then A is an intuitionistic L-fuzzy M-subgroup of G.

(ii) if $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, then B is an intuitionistic L-fuzzy M-subgroup of H.

(iii) either A is an intuitionistic anti L-fuzzy M-subgroup of G or B is an intuitionistic anti L-fuzzy M-subgroup of H, where e and e^{l} are the identity elements of G and H, respectively.

3.7 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. Thehomomorphic image of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G^{l}.

3.8 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. The homomorphic pre-image of an intuitionistic anti L-fuzzy M-subgroup of G^{l} is an intuitionistic anti L-fuzzy M-subgroup of G.

3.9 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. The anti-homomorphic image of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G^{l}.

3.10 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. The anti-homomorphic pre-image of an intuitionistic anti L-fuzzy M-subgroup of G^{l} is an intuitionistic anti L-fuzzy M-subgroup of G.

3.11 Theorem: Let A be an intuitionistic antiL-fuzzy M-subgroup of a M-group H and f is an isomorphism from a M-group G onto H. Then A∘f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

3.12 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H and f is an anti-isomorphism from a M-group G onto H. Then A∘f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

3.13 Theorem: Let (G, ·)be a M-group. If A and B are two intuitionistic anti L-fuzzy normal M-subgroups of G, then their intersection A∩B is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Proof: Let x and y in G and A = {⟨ x, $\mu_A(x)$, $\nu_A(x)$⟩ / x∈G } and B = { ⟨x, $\mu_B(x)$, $\nu_B(x)$⟩ / x∈G } be an intuitio3nistic anti L-fuzzy normal M-subgroups of G.

Let C = A∩B and C = { ⟨ x, $\mu_C(x)$, $\nu_C(x)$ ⟩ / x∈G }. Then, Clearly C is an intuitionistic anti L-fuzzy M-subgroup of a M –group G, since A and B are two intuitionistic anti L-fuzzy M-subgroups of a M-group G.Now, $\mu_C(xy) = \mu_A(xy) \vee \mu_B(xy) = \mu_A(yx) \vee \mu_B(yx) = \mu_C(yx)$.

Therefore, $\mu_C(xy) = \mu_C(yx)$, for all x and y in G. And,$\nu_C(xy) = \nu_A(xy) \wedge \nu_B(xy) = \nu_A(yx) \wedge \nu_B(yx) = \nu_C(yx)$.

Therefore, $\nu_C(xy) = \nu_C(yx)$, for all x and y in G. Hence A∩B is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

3.14 Theorem: Let (G, ·) be a M-group. The intersection of a family of intuitionistic antiL-fuzzy normal M-subgroups of G is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Proof: Let $\{ A_i \}_{i \in I}$ be a family of intuitionistic anti L-fuzzy normal M-subgroups of a M-group G and A = $\bigcap_{i \in I} A_i$. Then for x and y in G, clearly the intersection of a family of intuitionistic anti L-fuzzy M-subgroups of a M-group G is an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Now, $\mu_A(xy) = \sup_{i \in I} \mu_{A_i}(xy) = \sup_{i \in I} \mu_{A_i}(yx) = \mu_A(yx)$. Therefore, $\mu_A(xy)=\mu_A(yx)$, for all x and y in G. And, $\nu_A(xy) = \inf_{i \in I} \nu_{A_i}(xy) = \inf_{i \in I} \nu_{A_i}(yx) = \nu_A(yx)$. Therefore, $\nu_A(xy) = \nu_A(yx)$, for all x and y in G. Hence the intersection of a family of intuitionistic anti L-fuzzy normal M-subgroup of a

M-group G is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

3.15 Theorem: If A is an intuitionistic anti L-fuzzy characteristic M-subgroup of a M-group G, then A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

Proof: Let A be an intuitionistic anti L-fuzzy characteristic M-subgroup of a M-group G, x and y in G. Consider the map f : G → G defined by $f(x) = yxy^{-1}$. Clearly, f in AutG. Now, $\mu_A(xy) = \mu_A(f(xy))= \mu_A(y(xy)y^{-1})= \mu_A(yx)$. Therefore, $\mu_A(xy) = \mu_A(yx)$, for all x and y in G. Again, $\nu_A(xy) = \nu_A(f(xy)) = \nu_A(y(xy)y^{-1}) = \nu_A(yx)$. Therefore, $\nu_A(xy) =\nu_A(yx)$, for all x and y in G. Hence A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

3.16 Theorem : An intuitionistic anti L-fuzzy M-subgroup A of a M-group G is an intuitionistic anti L-fuzzy normal M-subgroup of G if and only if A is constant on the conjugate classes of G.

Proof : Suppose that A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G. Let x and y in G. Now, $\mu_A(y^{-1}xy) = \mu_A(xyy^{-1})= \mu_A(x)$. Therefore, $\mu_A(y^{-1}xy) = \mu_A(x)$, for all x and y in G. And, $\nu_A(y^{-1}xy) = \nu_A(xyy^{-1})= \nu_A(x)$. Therefore, $\nu_A(y^{-1}xy) = \nu_A(x)$, for all x and y in G. Hence $(x) = \{ y^{-1}xy / y\in G \}$. Hence A is constant on the conjugate classes of G. Conversely, suppose that A is constant on the conjugate classes of G. Then, $\mu_A(xy)= \mu_A(xyxx^{-1})= \mu_A(x(yx)x^{-1})= \mu_A(yx)$. Therefore, $\mu_A(xy) = \mu_A(yx)$, for all x and y in G. And, $\nu_A(xy)=\nu_A(xyxx^{-1}) = \nu_A(x(yx)x^{-1})= \nu_A(yx)$.Therefore, $\nu_A(xy) = \nu_A(yx)$, for all x and y in G. Hence A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

3.17 Theorem: Let A be an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G. Then for any y in G, we have $\mu_A(yxy^{-1}) = \mu_A(y^{-1}xy)$ and $\nu_A(yxy^{-1}) = \nu_A(y^{-1}xy)$, for every x in G.

Proof: Let A be an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G. For any y in G, we have, $\mu_A(yxy^{-1})= \mu_A(yy^{-1}x)= \mu_A(x)=\mu_A(xyy^{-1})= \mu_A(y^{-1}xy)$. Therefore, $\mu_A(yxy^{-1}) = \mu_A(y^{-1}xy)$, for all x and y in G. And, $\nu_A(yxy^{-1})=\nu_A(yy^{-1}x)= \nu_A(x)= \nu_A(xyy^{-1})= \nu_A(y^{-1}xy)$.

Therefore, $\nu_A(yxy^{-1})= \nu_A(y^{-1}xy)$, for all x & y in G.

3.18 Theorem: An intuitionistic anti L-fuzzy M-subgroup A of a M-group G is normalized if and only if $\mu_A(e) = 1$ and $\nu_A(e) = 0$, where e is the identity element of the M-group G.

Proof: If A is normalized, then there exists x in G such that $\mu_A(x) = 1$ and$\nu_A(x) = 0$, but by properties of an intuitionistic anti L-fuzzy M-subgroup A of the M-group G, $\mu_A(x) \leq \mu_A(e)$ and $\nu_A(x) \geq \nu_A(e)$, for every x in G. Since $\mu_A(x) = 1$ and $\nu_A(x) = 0$ and $\mu_A(x) \leq \mu_A(e)$ and $\nu_A(x) \geq \nu_A(e)$, $1 \leq \mu_A(e)$ and $0 \geq \nu_A(e)$. But$1 \geq \mu_A(e)$ and$0 \leq \nu_A(e)$. Hence $\mu_A(e) = 1$ and $\nu_A(e) = 0$. Conversely, if $\mu_A(e) = 1$ and $\nu_A(e) = 0$, then by the definition of normalized intuitionistic anti L-fuzzy subset, A is normalized.

3.19 Theorem: Let A and B be intuitionistic anti L-fuzzy M-subgroups of the M-groups G and H, respectively. If A and B are intuitionistic anti L-fuzzy normal M-subgroups, then AxB is an intuitionistic anti L-fuzzy normal M-subgroup of GxH.

Proof: Let A and B be intuitionistic anti L-fuzzy normal M-subgroups of the M-groups G and H respectively. Clearly AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH. Let x_1 and x_2 be in G, $y_1$and $y_2$be in H. Then (x_1, y_1) and (x_2, y_2) are in GxH. Now, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}(x_1x_2, y_1y_2)= \mu_A(x_1x_2) \wedge\mu_B(y_1y_2) = \mu_A(x_2x_1) \wedge\mu_B(y_2y_1)= \mu_{AxB}(x_2x_1, y_2y_1)= \mu_{AxB}[(x_2, y_2)(x_1, y_1)]$. Therefore, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}[(x_2, y_2)(x_1, y_1)]$, for all x_1, x_2 in G and y_1 and y_2 in H. And, $\nu_{AxB}[(x_1, y_1)(x_2, y_2)] = \nu_{AxB}(x_1x_2, y_1y_2)= \nu_A(x_1x_2) \vee\nu_B(y_1y_2) = \nu_A(x_2x_1) \vee\nu_B(y_2y_1)= \nu_{AxB}(x_2x_1, y_2y_1)= \nu_{AxB}[(x_2, y_2)(x_1, y_1)]$.

Therefore, $\nu_{AxB}[(x_1, y_1)(x_2, y_2)]=\nu_{AxB}[(x_2, y_2)(x_1, y_1)]$, for all x_1, x_2 in G and y_1 and y_2 in H. Hence AxB is an intuitionistic anti L-fuzzy normal M-subgroup of GxH.

3.20 Theorem: Let an intuitionistic anti L-fuzzy normal M-subgroup A of a M-group G be conjugate to an intuitionistic anti L-fuzzy normal M-subgroup M of G and an intuitionistic anti L-fuzzy normal M-subgroup B of a M-group H be conjugate to an intuitionistic anti L-fuzzy normal M-subgroup N of H. Then an intuitionistic anti L-fuzzy normal M-subgroup AxB of a M-group GxH is conjugate to an intuitionistic anti L-fuzzy normal M-subgroup MxN of GxH.

Proof: It is trivial.

3.21 Theorem: Let A and B be intuitionistic anti L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e^{I} are the identity element of G and H, respectively. If AxB is an intuitionistic anti L-fuzzy normal M-subgroup of GxH, then at least one of the following two statements must hold.

(i) $\mu_B(e^{I}) \leq \mu_A(x)$ and $\nu_B(e^{I}) \geq \nu_A(x)$, for all x in G,

(ii) $\mu_A(e) \leq \mu_B(y)$ and $\nu_A(e) \geq \nu_B(y)$, for all y in H.

Proof : It is trivial.

3.22 Theorem: Let A and B be intuitionistic anti L-fuzzy subsets of the M-groups G and H, respectively and AxB is an intuitionistic anti L-fuzzy normal M-subgroup of GxH. Then the following are true :

(i) if $\mu_A(x) \geq \mu_B(e^{I})$ and $\nu_A(x) \leq \nu_B(e^{I})$, then A is an intuitionistic L-fuzzy normal M-subgroup of G.

(ii) if $\mu_B(x) \geq \mu_A(e)$ and $\nu_B(x) \leq \nu_A(e)$, then B is an intuitionistic anti L-fuzzy normal M-subgroup of H.

(iii) either A is an intuitionistic anti L-fuzzy normal M-subgroup of G or B is an intuitionistic anti L-fuzzy normal M-subgroup of H.

Proof : It is trivial.

4. INTUITIONISTIC ANTI L-FUZZY NORMAL M-SUBGROUPS OF M-GROUPS UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM

4.1 Theorem: Let (G, ·) and (G^{I}, ·) be any two M-groups. The homomorphic image of an intuitionistic anti L-fuzzy normal M-subgroup of G is an intuitionistic anti L-fuzzy normal M-subgroup of G^{I}.

Proof: Let (G, ·) and (G^{I}, ·) be any two M-groups and $f : G \rightarrow G^{I}$ be a homomorphism. That is f(xy) = f(x)f(y), f(mx) = mf(x), for all x and y in G and m in M. Let V = f(A),where A is an intuitionistic anti L-fuzzy normal M-subgroup of G. We have to prove that V is an intuitionistic anti L-fuzzy normal M-subgroup of G^{I}. Now, for f(x) and f(y) in G^{I}, we have clearly V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G^{I}, since A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Now, $\mu_V(f(x)f(y)) = \mu_V(f(xy)) \leq \mu_A(xy) = \mu_A(yx) \geq \mu_V(f(yx)) = \mu_V(f(y)f(x))$, which implies that $\mu_V(f(x)f(y)) = \mu_V(f(y)f(x))$, for all xand y in G. Now, $\nu_V(f(x)f(y)) = \nu_V(f(xy)) \geq \nu_A(xy) = \nu_A(yx) \leq \nu_V(f(yx)) = \nu_V(f(y)f(x))$, which implies that $\nu_V(f(x)f(y)) = \nu_V(f(y)f(x))$, for all x and y in G. Hence V is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G^{I}.

4.2 Theorem: Let (G, ·) and (G^{I}, ·) be any two M-groups. The homomorphic pre-image of an intuitionistic anti L-fuzzy normal M-subgroup of G^{I} is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Proof: Let (G, ·) and (G^{I}, ·) be any two M-groups. Let $f : G \rightarrow G^{I}$ be a homomorphism. That is f(xy) = f(x)f(y), f(mx) = mf(x), for all x and y in G and m in M. Let V=f(A), where V is an intuitionistic anti L-fuzzy normal M-subgroup of G^{I}. We have to prove that A is an intuitionistic anti L-fuzzy normal M-subgroup of G. Let x and y in G. Then, clearly A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G, since V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G^{I}. Now, $\mu_A(xy) = \mu_V(f(xy)) = \mu_V(f(x)f(y)) = \mu_V(f(y)f(x)) = \mu_V(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in G. Now, $\nu_A(xy) = \nu_V(f(xy)) = \nu_V(f(x)f(y)) = \nu_V(f(y)f(x)) = \nu_V(f(yx)) = \nu_A(yx)$, which implies that $\nu_A(xy)= \nu_A(yx)$, for all x and y in G. Hence A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

4.3 Theorem: Let (G, ·) and (G^{I}, ·) be any two M-groups. The anti-homomorphic image of an intuitionistic anti L-fuzzy normal M-subgroup of G is an intuitionistic anti L-fuzzy normal M-subgroup of G^{I}.

Proof: Let (G, ·) and (G^{I}, ·) be any two M-groups. Let $f : G \rightarrow G^{I}$ be an anti-homomorphism. That is f(xy) = f(y)f(x), f(mx) = mf(x), for all x and y in G and m in M. Let V = f(A), where A is an intuitionistic anti L-fuzzy normal M-subgroup of G. We have to prove that V is an intuitionistic anti L-fuzzy normal M-subgroup of G^{I}.For f(x) and f(y) in G^{I}, clearly V is an intuitionistic anti L-fuzzy M-subgroup (IALFMSG) of a M-group G^{I}, since A is an intuitionistic anti L-fuzzy M-subgroup G.

Now, $\mu_V(f(x)f(y)) = \mu_V(f(yx)) \leq \mu_A(yx)=\mu_A(xy) \geq \mu_V(f(xy))= \mu_V(f(y)f(x))$, which implies that $\mu_V(f(x)f(y)) = \mu_V(f(y)f(x))$, for all x and y in G. And, $\nu_V(f(x)f(y)) = \nu_V(f(yx)) \geq \nu_A(yx) = \nu_A(xy) \leq \nu_V(f(xy))= \nu_V(f(y)f(x))$,

which implies that $\nu_V(f(x)f(y)) = \nu_V(f(y)f(x))$. Hence V is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G^{I}.

4.4 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. The anti-homomorphic pre-image of an intuitionistic anti L-fuzzy normal M-subgroup of G^{l} is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Proof: Let (G, ·) and (G^{l}, ·) be any two M-groups. Let $f : G \rightarrow G^{l}$ be an anti-homomorphism. That is f(xy) = f(y)f(x), f(mx) = mf(x), for all x and y in G and m in M. Let V=f(A) , where V is an intuitionistic anti L-fuzzy normal M-subgroup of G^{l}. We have to prove that A is an intuitionistic anti L-fuzzy normal M-subgroup of G.Let x and y in G, we have clearly A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G, since V is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}. Now, $\mu_A(xy) = \mu_V(f(xy)) = \mu_V(f(y)f(x)) = \mu_V(f(x)f(y)) = \mu_V(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in G. Now, $\nu_A(xy) = \nu_V(f(xy)) = \nu_V(f(y)f(x)) = \nu_V(f(x)f(y)) = \nu_V(f(yx)) = \nu_A(yx)$, which implies that $\nu_A(xy)=\nu_A(yx)$, for all x and y in G. Hence A is an intuitionisticantiL-fuzzy normal M-subgroup of a M-group G.

In the following Theorem ◦ is the composition operation of functions

4.5 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H and f is an isomorphism from a M-group G onto H.If A is an intuitionistic anti L-fuzzy normal M-subgroup of H, then A◦f is an intuitionisticanti L-fuzzy normal M-subgroup of a M-group G.

Proof: Let x and y in G and A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H. We know that, A◦f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Then we have, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) = \mu_A(f(y)f(x)) = \mu_A(f(yx)) = (\mu_A \circ f)(yx)$, which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$, for all x and y in G. Now, $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(x)f(y)) = \nu_A(f(y)f(x)) = \nu_A(f(yx)) = (\nu_A \circ f)(yx)$, which implies that $(\nu_A \circ f)(xy) = (\nu_A \circ f)(yx)$, for all x and y in G. Hence A◦f is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

4.6 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H and f is an anti-isomorphism from a M-group G onto H. If A is an intuitionistic anti L-fuzzy normal M-subgroup of H, then A◦f is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

Proof: Let x and y in G and A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H. We know that, A◦f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G. Then we have, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) = \mu_A(f(x)f(y)) = \mu_A(f(yx)) = (\mu_A \circ f)(yx)$, which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$, for all x and y in G. Now, $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x)) = \nu_A(f(x)f(y)) = \nu_A(f(yx)) = (\nu_A \circ f)(yx)$, which implies that$(\nu_A \circ f)(xy) = (\nu_A \circ f)(yx)$, for all x and y in G. Hence (A◦f) is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

V CONCLUSION

Further work is in progress in order to develop theintuitionistic anti L- fuzzy normal M -subgroups, intuitionisticanti L- fuzzy N-subgroup and intuitionistic anti L-fuzzy normalN-subgroups.

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