A Study on Intuitionistic Anti L-Fuzzy M-Subgroups

P. Pandiammal
Professor, Department of Mathematics,
PSNA College of Engg., & Tech., Dindigul, Tamil Nadu, India

***Abstract*–**In this paper, we introduce the concept of intuitionistic anti L-fuzzy M- subgroups and investigate some related properties.

***Keywords*:** Intuitionistic fuzzy subsets; Intuitionistic anti fuzzy subgroups; Intuitionistic anti L-fuzzy M-subgroups; Intuitionistic anti fuzzy characteristic; M-homomorphism.

I. INTRODUCTION

A fuzzy set theory has developed in many directions and finding application in a wide variety of fields. Zadeh's classical paper [19] of 1965 introduced the concepts of fuzzy sets and fuzzy set operations. The study of fuzzy groups was started by Rosenfeld [15] and it was extended by Roventa [16] who have introduced the concept of fuzzy groups operating on fuzzy sets and many researchers [1,7,9,10] are engaged in extending the concepts. The concept of intuitionistic fuzzy set was introduced by Atanassov. K.T [2,3], as a generalization of the notion of fuzzy sets. Choudhury. F.P et al [6] defined a fuzzy subgroup and fuzzy homomorphism. **Palaniappan. N** and **Muthuraj,** [11] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. **Pandiammal. P, Natarajan. R** and **Palaniappan. N,** [13] defined the homomorphism, anti-homomorphism of an anti L-fuzzy M-subgroup.In this paper we introduce and discuss the algebraic nature of intuitionistic anti L-fuzzy M-groups with operator and obtain some related results.

II. PRELIMINARIES

2.1 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be **anti L-fuzzy M-subgroup** (ALFMSG) of G if its satisfies the following axioms:

(i) $\mu_A(\,mxy\,) \le \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(\,x^{-1}\,) \le \mu_A(\,x\,)$, for all x and y in G.

2.2 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an **intuitionistic L-fuzzy M-subgroup (ILFMSG)** of G if the following conditions are satisfied:

(i) $\mu_A(\,mxy\,) \ge \mu_A(x) \wedge \mu_A(y)$,

(ii) $\mu_A(x^{-1}) \ge \mu_A(x)$,

(iii) $\nu_A(\,mxy\,) \le \nu_A(x) \vee \nu_A(y)$,

(iv) $\nu_A(\,x^{-1}\,) \le \nu_A(\,x\,)$, for all x & y in G.

2.3 Definition: Let (G, ·) and (G^{l}, ·) be any two M-groups. Let $f : G \to G^{l}$ be any function and A be an intuitionistic L-fuzzy M-subgroup in G, V be an intuitionistic L-fuzzy M-subgroup in f (G) = G^{l}, defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x \in f^{-1}(y)} \nu_A(x)$, for all x in G and y in G^{l}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2.4 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as $AxB = \{ \langle (x, y), \mu_{AxB}(x, y), \nu_{AxB}(x, y) \rangle$ / for all x in G and y in H }, where $\mu_{AxB}(\,x, y\,) = \mu_A(x) \wedge \mu_B(y)$ and $\nu_{AxB}(\,x, y\,) = \nu_A(x) \vee \nu_B(y)$.

2.5 Definition: Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **conjugate intuitionistic L-fuzzy M-subgroups** of G if for some g in G, $\mu_A(x) = \mu_B(\,g^{-1}xg\,)$ and $\nu_A(x) = \nu_B(\,g^{-1}xg\,)$, for every x in G.

2.6 Definition: Let A be an intuitionistic L-fuzzy subset in a set S, the **strongest intuitionistic L-fuzzy relation** on S, that is an intuitionistic L-fuzzy relation on A is V given by $\mu_V(\,x, y\,) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_V(x, y\,) = \nu_A(x) \vee \nu_A(y)$, for all x and y in S.

III. INTUITIONISTIC ANTI L-FUZZY M-SUBGROUPS

3.1 Defintion: An intuitionistic fuzzy subset μ in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following axioms are satisfied.

(i) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(x^{-1}) \leq \mu_A(x)$,

(iii) $\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$,

(iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x and y in G.

3.2 Proposition: Let G be a group. An intuitionistic fuzzy subset μ in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following conditions are satisfied.(i) $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, (ii) $\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$, for all x ,y in G.

3.3 Definition: Let G be an M-group and μ be an intuitionistic anti fuzzy group of G. If $\mu_A(mx) \leq \mu_A(x)$ and $\nu_A(mx) \geq \nu_A(x)$ for all x in G and m in M then μ is said to be an intuitionistic anti fuzzy subgroup with operator of G. We use the phrase μ is an intuitionistic anti L-fuzzy M-subgroup of G.

3.4 Example: Let H be M-subgroup of an M-group G and let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in G defined by

$$\mu_A(x) = \begin{cases} 0.3; & x \in H \\ 0.5; & \text{otherwise} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.6; & x \in H \\ 0.3; & \text{otherwise} \end{cases}$$

For all x in G. Then it is easy to verify that $A = (\mu_A, \nu_A)$ is an anti fuzzy M- subgroup of G.

3.5 Definition: Let A and B be any two intuitionistic anti L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be **conjugate intuitionistic anti L-fuzzy M-subgroups** of G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$ and $\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

3.6 Proposition: If μ = (δμ , λμ) is an intuitionistic anti fuzzy M-subgroup of an M- group G, then for any x, y∈ G and m ∈ M.

(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(mx^{-1}) \leq \mu_A(x)$ and

(iii) $\nu_A(mxy) \geq \nu_A(x) \wedge \nu_A(y)$,

(iv) $\nu_A(mx^{-1}) \leq \nu_A(x)$, for all x and y in G.

3.7 Theorem: A is an intuitionistic anti L-fuzzy M-subgroup of a M-group (G, ·) if and only if $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Proof: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group (G, ·).

Then, $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y^{-1})$

$\leq \mu_A(x) \vee \mu_A(y)$, since A is an IALFMSG of G.

Therefore, $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

And, $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y^{-1})$

$\geq \nu_A(x) \wedge \nu_A(y)$, since A is an IALFMSG of G.

Therefore, $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Conversely, if $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$,

replace y by x, then, $\mu_A(x) \geq \mu_A(e)$ and $\nu_A(x) \leq \nu_A(e)$, for all x and y in G.

Now, $\mu_A(x^{-1}) = \mu_A(ex^{-1})$

$\leq \mu_A(e) \vee \mu_A(x) = \mu_A(x)$.

Therefore, $\mu_A(x^{-1}) \leq \mu_A(x)$.

It follows that, $\mu_A(xy) = \mu_A(x(y^{-1})^{-1})$

$\leq \mu_A(x) \vee \mu_A(y^{-1})$

$\leq \mu_A(x) \vee \mu_A(y)$.

Therefore, $\mu_A(xy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

And, $\nu_A(x^{-1}) = \nu_A(ex^{-1})$

$\geq \nu_A(e) \wedge \nu_A(x)$

$= \nu_A(x)$.

Therefore, $\nu_A(x^{-1}) \geq \nu_A(x)$.

Then, $\nu_A(xy) = \nu_A(x(y^{-1})^{-1}) \geq \nu_A(x) \wedge \nu_A(y^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$.

Therefore, $\nu_A(xy) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Hence A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

3.8 Theorem: Let A be an intuitionistic anti L-fuzzy subset of a group (G, ·). If $\mu_A(e) = 0$ and $\nu_A(e) = 1$ and $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y)$, , for all x and y in G, then A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

Proof: Let x and y in G and e is the identity element in G.

Now, $\mu_A(x^{-1}) = \mu_A(ex^{-1}) \leq \mu_A(e) \vee \mu_A(x) = 0 \vee \mu_A(x) = \mu_A(x)$

Therefore, $\mu_A(x^{-1}) \leq \mu_A(x)$, for all x in G.

And $\nu_A(x^{-1}) = \nu_A(ex^{-1})$

$\geq \nu_A(e) \wedge \nu_A(x)$

$= 0 \wedge \nu_A(x)$

$= \nu_A(x)$.

Therefore, $\nu_A(x^{-1}) \geq \nu_A(x)$, for all x in G.

Now, $\mu_A(mxy) = \mu_A(x(y^{-1})^{-1})$

$\leq \mu_A(x) \vee \mu_A(y^{-1})$

$\leq \mu_A(x) \vee \mu_A(y)$.

Therefore, $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

And, $\nu_A(mxy) = \nu_A(x(y^{-1})^{-1})$

$\geq \nu_A(x) \wedge \nu_A(y^{-1})$

$\geq \nu_A(x) \wedge \nu_A(y)$.

Therefore, $\nu_A(mxy) \geq \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

Hence A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

2.9 Theorem: If A is an intuitionistic anti L-fuzzy M-subgroup of a M-group (G, ·), then $H = \{x / x \in G : \mu_A(x) = 0, \nu_A(x) = 1\}$ is either empty or is a M-subgroup of a M-group G.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y^{-1}) \leq \mu_A(x) \vee \mu_A(y) = 0$.

Therefore, $\mu_A(xy^{-1}) = 0$, for all x and y in G.

And, $\nu_A(xy^{-1}) \geq \nu_A(x) \wedge \nu_A(y^{-1})$

$= \nu_A(x) \wedge \nu_A(y)$, (since A is an IALFMSG of G)

$= 1 \wedge 1 = 1$.

Therefore, $\nu_A(xy^{-1}) = 1$, for all x and y in G. We get mxy^{-1} in H.

Therefore, H is a M-subgroup of a M-group G.

Hence H is either empty or is a M-subgroup of M-group G.

2.10 Theorem: If A is an intuitionistic anti L-fuzzy M-subgroup of a M-group (G, ·), then $H = \{ x \in G: \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)\}$ is either empty or is a M-subgroup of a M-group G.

Proof: If no element satisfies this condition, then H is empty.

If x and y satisfies this condition, then $\mu_A(x^{-1}) = \mu_A(x) = \mu_A(e)$, $\nu_A(x^{-1}) = \nu_A(x) = \nu_A(e)$

Therefore, $\mu_A(x^{-1}) = \mu_A(e)$ and $\nu_A(x^{-1}) = \nu_A(e)$.

Hence x^{-1} in H.

Now, $\mu_A(mxy^{-1}) \leq \mu_A(x) \vee \mu_A(y^{-1})$

$\leq \mu_A(x) \vee \mu_A(y)$

$= \mu_A(e) \vee \mu_A(e) = \mu_A(e)$.

Therefore, $\mu_A(mxy^{-1}) \leq \mu_A(e)$, for all x and y in G---(1).

And, $\mu_A(e) = \mu_A((xy^{-1})(xy^{-1})^{-1})$

$\leq \mu_A(xy^{-1}) \vee \mu_A((xy^{-1})^{-1})$

$\leq \mu_A(xy^{-1}) \vee \mu_A(xy^{-1})$

$= \mu_A(xy^{-1})$.

Therefore, $\mu_A(e) \leq \mu_A(xy^{-1})$, for all x and y in G----(2).

From (1) and (2), we get $\mu_A(e) = \mu_A(xy^{-1})$.

Now, $\nu_A(mxy^{-1}) \geq \nu_A(x) \wedge \nu_A(y^{-1})$

$\geq \nu_A(x) \wedge \nu_A(y)$

$= \nu_A(e) \wedge \nu_A(e) = \nu_A(e)$.

Therefore, $\nu_A(mxy^{-1}) \geq \nu_A(e)$, for all x and y in G----(3).

And, $\nu_A(e) = \nu_A((xy^{-1})(xy^{-1})^{-1})$

$\geq \nu_A(xy^{-1}) \wedge \nu_A((xy^{-1})^{-1})$

$\geq \nu_A(xy^{-1}) \wedge \nu_A(xy^{-1})$

$= \nu_A(xy^{-1})$.

Therefore, $\nu_A(e) \geq \nu_A(xy^{-1})$, for all x and y in G-----(4).

From (3) and (4), we get $\nu_A(e) = \nu_A(xy^{-1})$.

Hence $\mu_A(e) = \mu_A(xy^{-1})$ and $\nu_A(e) = \nu_A(xy^{-1})$.

Therefore, mxy^{-1} in H.

Hence H is either empty or is a M-subgroup of a M-group G.

2.11 Theorem: Let (G, ·) be a M-group. If A is an intuitionistic anti L-fuzzy M-subgroup of G, then $\mu_A(xy) = \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) = \nu_A(x) \wedge \nu_A(y)$ with $\mu_A(x) \neq \mu_A(y)$ and $\nu_A(x) \neq \nu_A(y)$, for each x and y in G.

Proof: Let x and y belongs to G.

Assume that $\mu_A(x) < \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$.

Now, $\mu_A(y) = \mu_A(x^{-1}xy)$

$\leq \mu_A(x^{-1}) \vee \mu_A(xy)$

$\leq \mu_A(x) \vee \mu_A(xy)$

$= \mu_A(xy)$

$\leq \mu_A(x) \vee \mu_A(y)$

$= \mu_A(y)$.

Therefore, $\mu_A(xy) = \mu_A(y) = \mu_A(x) \vee \mu_A(y)$, for all x and y in G.

And, $\nu_A(y) = \nu_A(x^{-1}xy)$

$\geq \nu_A(x^{-1}) \wedge \nu_A(xy)$

$\geq \nu_A(x) \wedge \nu_A(xy) = \nu_A(xy)$

$\geq \nu_A(x) \wedge \nu_A(y) = \nu_A(y)$.

Therefore, $\nu_A(xy) = \nu_A(y) = \nu_A(x) \wedge \nu_A(y)$, for all x and y in G.

2.12 Theorem: If A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G, then (i) $\mu_A(xy) = \mu_A(yx)$ if and only if $\mu_A(x) = \mu_A(y^{-1}xy)$, (ii) $\nu_A(xy) = \nu_A(yx)$ if and only if $\nu_A(x) = \nu_A(y^{-1}xy)$, for x and y in G.

Proof: Let x and y be in G.

Assume that $\mu_A(xy) = \mu_A(yx)$, we have

$\mu_A(y^{-1}xy) = \mu_A(y^{-1}yx) = \mu_A(ex) = \mu_A(x)$.

Therefore, $\mu_A(x) = \mu_A(y^{-1}xy)$, for all x and y in G.

Conversely, assume that $\mu_A(x) = \mu_A(y^{-1}xy)$,

we have $\mu_A(xy) = \mu_A(xyxx^{-1}) = \mu_A(yx)$.

Therefore, $\mu_A(xy) = \mu_A(yx)$, for all x and y in G.

Hence (i) is proved

Now, we assume that $\nu_A(xy) = \nu_A(yx)$,

we have $\nu_A(y^{-1}xy) = \nu_A(y^{-1}yx) = \nu_A(ex) = \nu_A(x)$.

Therefore, $\nu_A(x) = \nu_A(y^{-1}xy)$, for all x and y in G.

Conversely, we assume that $\nu_A(x) = \nu_A(y^{-1}xy)$,

we have $\nu_A(xy) = \nu_A(xyxx^{-1}) = \nu_A(yx)$.

Therefore, $\nu_A(xy) = \nu_A(yx)$, for all x and y in G.

Hence (ii) is proved.

2.13 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group G such that Im $\mu_A = \{ \alpha \}$ and Im $\nu_A = \{ \beta \}$, where α and β in L.

If $A = B \cup C$, where B and C are intuitionistic anti L-fuzzy M-subgroups of G, then either $B \subseteq C$ or $C \subseteq B$.

Proof:

Case (i) : Let $A = B \cup C = \{ \langle x , \mu_A(x) , \nu_A(x) \rangle / x \in G \}$,

$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in G \}$ and $C = \{ \langle x, \mu_C(x), \nu_C(x) \rangle / x \in G \}$.

Assume that $\mu_B(x) < \mu_C(x)$ and $\mu_B(y) > \mu_C(y)$, for some x and y in G.

Then, $\alpha = \mu_A(x) = \mu_{B\cup C}(x) = \mu_B(x) \wedge \mu_C(x) = \mu_B(x) > \mu_C(x)$.

Therefore, $\alpha > \mu_C(x)$, for all x in G.

And, $\alpha = \mu_A(y) = \mu_{B\cup C}(y) = \mu_B(y) \wedge \mu_C(y) = \mu_C(y) > \mu_B(y)$.

Therefore, $\alpha > \mu_B(y)$, for all y in G.So that, $\mu_C(y) > \mu_C(x)$ and $\mu_B(x) > \mu_B(y)$.

Hence $\mu_B(xy) = \mu_B(y)$ and $\mu_C(xy) = \mu_C(x)$.

But then, $\alpha = \mu_A(xy) = \mu_{B\cup C}(xy) = \mu_B(xy) \vee \mu_C(xy) = \mu_B(y) \vee \mu_C(x) < \alpha$ --------------(1).

Case (ii): Assume that $\nu_B(x) < \nu_C(x)$ and $\nu_B(y) > \nu_C(y)$, for some x and y in G.

Then, $\beta = \nu_A(x) = \nu_{B\cup C}(x) = \nu_B(x) \vee \nu_C(x) = \nu_B(x) < \nu_C(x)$. Therefore, $\beta < \nu_C(x)$, for all x in G.

And, $\beta = \nu_A(y) = \nu_{B\cup C}(y) = \nu_B(y) \vee \nu_C(y) = \nu_C(y) < \nu_B(y)$.

Therefore, $\beta < \nu_B(y)$, for all x in G. So that, $\nu_C(y) < \nu_C(x)$ and $\nu_B(x) < \nu_B(y)$.

Hence $\nu_B(xy) = \nu_B(y)$ and $\nu_C(xy) = \nu_C(x)$.

But then, $\beta = \nu_A(xy) = \nu_{B\cup C}(xy) = \nu_B(xy) \vee \nu_C(xy) = \nu_B(y) \vee\nu_C(x) > \beta$ --------------(2).

It is a contradiction by (1) and (2).

Therefore, either $B \subseteq C$ or $C \subseteq B$ is true.

2.14 Theorem: If A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G and if there is a sequence $\{x_n\}$ in G such that $\lim_{n\to\alpha} \{ \mu_A(x_n) \vee \mu_A(x_n) \}=0$ and $\lim_{n\to\alpha} \{ \nu_A(x_n) \wedge \nu_A(x_n) \} = 1$, then $\mu_A(e) = 0$ and $\nu_A(e) = 1$, where e is the identity element in G.

Proof: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G with e as its identity element in G and x in G be an arbitrary element. We have x in G implies x^{-1} in G and hence $xx^{-1} = e$.

Then, we have $\mu_A(e) = \mu_A(xx^{-1}) \le \mu_A(x) \vee \mu_A(x^{-1}) \le \mu_A(x) \vee \mu_A(x)$.

For each n, we have $\mu_A(e) \le \mu_A(x) \vee\mu_A(x)$.

Since $\mu_A(e) \le \lim_{n\to\alpha} \{ \mu_A(x_n) \vee \mu_A(x_n) \} = 0$

Therefore $\mu_A(e) = 0$

And, $\nu_A(e) = \nu_A(xx^{-1}) \ge \nu_A(x) \wedge \nu_A(x^{-1}) = \nu_A(x) \wedge \nu_A(x)$.

For each n, we have $\nu_A(e) \ge \nu_A(x) \wedge \nu_A(x)$. Since $\nu_A(e) \ge \lim_{n\to\alpha} \nu_A(x_n) \wedge \nu_A(x_n) = 1$

Therefore, $\nu_A(e) = 1$

2.15 Theorem: If A and B are intuitionistic anti L-fuzzy M-subgroups of the M-groups G and H, respectively, then AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH.

Proof: Let A and B be intuitionistic anti L-fuzzy M-subgroups of the M-groups G and H respectively. Let x_1 and x_2 be in G, y_1 and y_2 be in H.

Then (x_1, y_1) and (x_2, y_2) are in GxH.

Now,

$\mu_{AxB} [m(x_1, y_1)(x_2, y_2)] = \mu_{AxB} (mx_1x_2, my_1y_2) = \mu_A(mx_1x_2) \vee \mu_B(my_1y_2) \le \{\mu_A(x_1) \vee \mu_A(x_2)\} \vee\{\mu_B(y_1) \vee \mu_B(y_2) \} = \{\mu_A(x_1) \vee \mu_B(y_1) \}\vee \{\mu_A(x_2) \vee\mu_B(y_2) \} = \mu_{AxB} (x_1, y_1) \vee \mu_{AxB} (x_2, y_2)$.

Therefore, $\mu_{AxB} [m(x_1, y_1)(x_2, y_2)] \le \mu_{AxB} (x_1, y_1) \vee \mu_{AxB} (x_2, y_2)$, for all x_1 and x_2 in G, y_1 and y_2 in H.

And, $\nu_{AxB} [m(x_1, y_1)(x_2, y_2)]$

$= \nu_{AxB} (mx_1x_2, my_1y_2)$

$= \nu_A(mx_1x_2) \wedge \nu_B(my_1y_2)$

$\geq \{\nu_A(x_1) \wedge \nu_A(x_2)\} \wedge \{\nu_B(y_1) \wedge \nu_B(y_2)\}$

$= \{\nu_A(x_1) \wedge \nu_B(y_1)\} \wedge \{\nu_A(x_2) \wedge \nu_B(y_2)\}$

$= \nu_{AxB}(x_1, y_1) \wedge \nu_{AxB}(x_2, y_2)$.

Therefore, $\nu_{AxB}[\, m(x_1, y_1)(x_2, y_2)\,] \geq \nu_{AxB}(x_1, y_1) \wedge \nu_{AxB}(x_2, y_2)$, for all x_1 and x_2 in G, y_1 and y_2 in H. Hence AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH.

2.16 Theorem: Let an intuitionistic anti L-fuzzy M-subgroup A of a M-group G be conjugate to an intuitionistic anti L-fuzzy M-subgroup M of G and an intuitionistic anti L-fuzzy M-subgroup B of a M-group H be conjugate to an intuitionistic anti L-fuzzy M-subgroup N of H. Then an intuitionistic anti L-fuzzy M-subgroup AxB of a M-group GxH is conjugate to an intuitionistic anti L-fuzzy M-subgroup MxN of GxH.

Proof: Let A and B be intuitionistic anti L-fuzzy M-subgroups of the M-groups G and H respectively. Let x, x^{-1} and f be in G and y, y^{-1} and g be in H.

Then (x, y) ,(x^{-1}, y^{-1}) and (f, g) are in GxH.

Now, $\mu_{AxB}(f, g) = \mu_A(f) \vee \mu_B(g)$

$= \mu_M(xf x^{-1}) \vee \mu_N(yg y^{-1})$

$= \mu_{MxN}(xf x^{-1}, yg y^{-1})$

$= \mu_{MxN}[\,(x, y)(f, g)(x^{-1}, y^{-1})\,]$

$= \mu_{MxN}[\,(x, y)(f, g)(x, y)^{-1}\,]$.

Therefore, $\mu_{AxB}(f, g) = \mu_{MxN}[\,(x, y)(f, g)(x, y)^{-1}\,]$, for all x, x^{-1} and f in G and y, y^{-1} and g in H.

And, $\nu_{AxB}(f, g) = \nu_A(f) \wedge \nu_B(g)$

$= \nu_M(xf x^{-1}) \wedge \nu_N(yg y^{-1})$

$= \nu_{MxN}(xf x^{-1}, yg y^{-1})$

$= \nu_{MxN}[\,(x, y)(f, g)(x^{-1}, y^{-1})\,]$

$= \nu_{MxN}[\,(x, y)(f, g)(x, y)^{-1}\,]$.

Therefore, $\nu_{AxB}(f, g) = \nu_{MxN}[\,(x, y)(f, g)(x, y)^{-1}\,]$, for all x, x^{-1} and f in G and y, y^{-1} and g in H.

Hence an intuitionistic anti L-fuzzy M-subgroup AxB of GxH is conjugate to an intuitionistic anti L-fuzzy M-subgroup MxN of GxH.

2.17 Theorem: Let A and B be intuitionistic L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e^{l} are the identity element of G and H, respectively. If AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH, then at least one of the following two statements

(i) $\mu_B(e^{l}) \leq \mu_A(x)$ and $\nu_B(e^{l}) \geq \nu_A(x)$, for all x in G,

(ii) $\mu_A(e) \leq \mu_B(y)$ and $\nu_A(e) \geq \nu_B(y)$, for all y in H.

Proof: Let AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH.

By contraposition, suppose that none of the statements (i) and (ii) holds.

Then we can find a in G and b in H such that

$\mu_A(a) < \mu_B(e^{l})$, $\nu_A(a) > \nu_B(e^{l})$ and $\mu_B(b) < \mu_A(e)$, $\nu_B(b) > \nu_A(e)$.

We have, $\mu_{AxB}(a, b) = \mu_A(a) \vee \mu_B(b)$

$< \mu_A(e) \vee \mu_B(e^{l}) = \mu_{AxB}(e, e^{l})$.

And, $\nu_{AxB}(a, b) = \nu_A(a) \wedge \nu_B(b)$

$> \nu_A(e) \wedge \nu_B(e^{l}) = \nu_{AxB}(e, e^{l})$.

Thus AxB is not an intuitionistic anti L-fuzzy M-subgroup of GxH.

Hence either $\mu_B(e^{l}) \leq \mu_A(x)$ and $\nu_B(e^{l}) \geq \nu_A(x)$, for all x in G or $\mu_A(e) \leq \mu_B(y)$ and $\nu_A(e) \geq \nu_B(y)$, for all y in H.

III - INTUITIONISTIC ANTI L-FUZZY M-SUBGROUPS UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM

3.1 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. The homomorphic image (pre-image) of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G^{l}.

Proof: Let (G, ·) and (G^{l}, ·) be any two groups and $f : G \rightarrow G^{l}$ be a homomorphism.

That is f(xy) = f(x)f(y), f(mx) = mf(x), for all x and y in G and m in M. Let V=f(A), where A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G. We have to prove that V is an intuitionistic anti L-fuzzy M-subgroup of G^{l}.

Now, for f(x) and f(y) in G^{l}, we have

$\mu_V(mf(x)f(y))=\mu_V(f(mxy))$, (as f is a homomorphism)

$\leq \mu_A(mxy)$

$\leq \mu_A(x) \vee \mu_A(y)$, as A is an IALFMSG of G

which implies that $\mu_V(mf(x)f(y)) \leq \mu_V(f(x)) \vee \mu_V(f(y))$, for all x and y in G.

For f(x) in G^{l}, we have,

$\mu_V([f(x)]^{-1})=\mu_V(f(x^{-1}))$, (as f is a homomorphism)

$\leq \mu_A(x^{-1})$

$\leq \mu_A(x)$, as A is an IALFMSG

which implies that $\mu_V([f(x)]^{-1}) \leq \mu_V(f(x))$, for all x in G.

$\nu_V(mf(x)f(y)) = \nu_V(f(mxy))$,

as f is a homomorphism

$\geq \nu_A(mxy)$

$\geq \nu_A(x) \wedge \nu_A(y)$,

as A is an IALFMSG of G,

which implies that $\nu_V(f(x)f(y)) \geq \nu_V(f(x)) \wedge \nu_V(f(y))$, for all x and y in G.

$\nu_V([f(x)]^{-1}) = \nu_V(f(x^{-1}))$,

(as f is a homomorphism)

$\geq \nu_A(x^{-1})$

$\geq \nu_A(x)$, as A is an IALFMSG of G,

which implies that $\nu_V([f(x)]^{-1}) \geq \nu_V(f(x))$, for all x in G.

Hence V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G^{l}.

3.2 Theorem: Let (G, ·) and (G^{l}, ·) be any two M-groups. The anti-homomorphic image (pre-image) of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G^{l}.

Proof: Let (G, ·) and (G^{l}, ·) be any two M-groups and f : G → G^{l} be an anti-homomorphism. That is f(xy) = f(y)f(x), f(mx) = m f(x), for all x and y in G and m in M. Let V = f(A), where A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

We have to prove that V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G^{l}.

Now, let f(x) and $f(y) \in G^{l}$, we have

$\mu_V(mf(x)f(y)) = \mu_V(f(myx))$,

(as f is an anti-homomorphism)

$\leq \mu_A(myx)$

$\leq \mu_A(x) \vee \mu_A(y)$,

(as A is an IALFMSG of G)

which implies that $\mu_V(mf(x)f(y)) \leq \mu_V(f(x)) \vee \mu_V(f(y))$, for all x and y in G.

For x in G, $\mu_V([f(x)]^{-1}) = \mu_V(f(x^{-1}))$

$\leq \mu_A(x^{-1}) \leq \mu_A(x)$, as A is an IALFMSG of G,

which implies that $\mu_V([f(x)]^{-1}) \leq \mu_V(f(x))$, for all x in G.

And, $\nu_V(mf(x)f(y)) = \nu_V(f(myx))$

$\geq \nu_A(myx) \geq \nu_A(x) \wedge \nu_A(y)$, as A is an IALFMSG of G,

which implies that $\nu_V(f(x)f(y)) \geq \nu_V(f(x)) \wedge \nu_V(f(y))$, for all x and y in G.

Also, $\nu_V([f(x)]^{-1}) = \nu_V(f(x^{-1}))$

$\geq \nu_A(x^{-1}) \geq \nu_A(x)$, as A is an IALFMSG of G,

which implies that $\nu_V([f(x)]^{-1}) \geq \nu_V(f(x))$, for all x in G.

Hence V is an intuitionistic anti L-fuzzy M-subgroup of a M-group $G^{|}$.

IV CONCLUSION

Further work is in progress in order to develop the intuitionistic anti L- fuzzy normal M -subgroups, intuitionistic anti L- fuzzy M-N-subgroup and intuitionistic anti L-fuzzy normal M-N-subgroups.

REFERENCES

[1] Aktas, H and Cagman, N., "Generalized product of fuzzy subgroup and t – level subgroups", Math. Commun, Vol. 11, pp.121– 128, 2006.

[2] Atanassov. K. T.," Intuitionistic fuzzy sets", Fuzzy sets and systems, Vol. 20, pp. 87 – 96,1986

[3] Atanassov. K. T., "New operations defined over the intuitionistic fuzzy sets", Fuzzy sets and systems, Vol. 61, pp.137 –142, 1994

[4] Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications 35, (1971) 512-517.

[5] R. Biswas, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy Sets and Systems 35 (1990), 121-124.

[6] Choudhury. F. P and Chakraborty. A. B and Khare. S. S., "A note on fuzzy subgroups and fuzzy homomorphism", Journal of mathematical analysis and applications, Vol.131, pp.537 – 553,1988.

[7] J.A. Goguen, L-fuzzy sets, J. Math. Anal. Appl.18(1967),145-179.

[8] Jacobson. N., Lectures in abstract algebras. East – west press, 1951.

[9] Mohamed Asaad, Groups and Fuzzy Subgroups, Fuzzy Sets and Systems 39(1991)323-328.

[10] Massa'deh. M. O and For a. A.A., "Centralizer of upper fuzzy groups", Pioneer journal of mathematics and mathematical sciences, Vol.1, pp. 81-88, 2011

[11] N.Palaniappan, R.Muthuraj, The homomorphism, Anti-homomorphism of a fuzzy and an anti-fuzzy group, Varahmihir Journal of mathematical Sciences, 4 (2)(2004) 387-399.

[12] N.Palaniappan , S. Naganathan, & K. Arjunan, A Study on Intuitionistic L-Fuzzy Subgroups, Applied mathematical Sciences, 3 (53) (2009) 2619-2624.

[13] Pandiammal. P, Natarajan. R, and Palaniappan. N , Anti L-fuzzy M- subgroups, Antarctica J. Math., Vol. 7, number 6, 683-691, (2010).

[14] Prabir Bhattacharya, Fuzzy subgroups: Some characterizations, J. Math. Anal. Appl. 128 (1981) 241-252.

[15] Rosenfeld. A.," Fuzzy groups", J. Math. Anal. Appl, Vol. 35, pp. 512 – 517, 1971.

[16] Roventa. A and Spircu. T.," Groups operating on fuzzy sets", Fuzzy sets and systems, Vol. 120 , pp. 543 – 548,2001.

[17] Sulaiman. R and Ahmad. A. G.," The number of fuzzy subgroups of finite ncyclic groups", Int. Math. Forum, Vol.6 , pp. 987 – 994,2011.

[18] Tarnauceanu. M and Bentea.L.," On the number of fuzzy subgroup of finite abelian groups", Fuzzy sets and systems, Vol. 159, pp. 1084 – 1096,2008.

[19] L.A. Zadeh, Fuzzy sets, Information and control, 8, (1965) 338-353.