FIR Filter Design With Farrow Structure Using Genetic Algorithm
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ABSTRACT

This paper proposes method to design variable fractional-delay (FD) filters using the Farrow structure. In the transfer function of the Farrow structure, different sub filters are weighted by different powers of the FD value. The variable fractional delay (VFD) digital filters as an important class of the variable digital filters have been receiving increasingly attention in the past decade. Under tuning a controlling parameter, this kind of filters changes continuously a delay, which is a fraction of the sampling period. VFD filters have many applications in different areas of signal processing and communication, for example, time adjustment in digital receivers, speech coding and synthesis, time delay estimation and analog–digital (A/D) conversion, etc. A method for developing VFD filters is also an essential technique for the fractional linear discrete-time systems. Theoretically speaking, the design of variable digital filters under optimal sense is more complicated and difficult than the design of fixed delay filters, since the impulse response or the poles and zeros of the filters are some type of functions in the variable parameter are generally assumed to be polynomial functions. Therefore sub optimal approaches for the design of variable digital filters should be investigated for the purpose of reducing the computation complexity. For instance the two-stage approach, i.e. designing a set of fixed-coefficient filters and then fitting each of the coefficients as polynomials has been proposed. Recently advances have been made on the design of some type of VFD filters, such as finite-impulse response (FIR) VFD filters and infinite-impulse response (IIR) all pass VFD filters. Large numbers of coefficients should be designed; related iteration algorithms still feature considerable computation complexity. Examples illustrate our proposed method and Comparisons, to various earlier designs show a reduction of the arithmetic complexity.

Keywords:
Farrow structure, Fractional-delay

1. Introduction

Digital Filters are used in numerous applications from control systems, systems for audio and video processing and communication systems to systems for medical applications to name just a few. They can be implemented in hardware or software and can process both real-time and offline (recorded) signals. Digital Filters in hardware form can now routinely perform tasks that were almost exclusively performed by analog systems in the past whereas software digital filters can be implemented using low-level or user-friendly high-level Programming languages. Beside the inherent advantages such as high accuracy and reliability, small physical size, and reduced sensitivity to component tolerances or drift, digital implementations allow one to achieve certain characteristics not possible with analog implementations such as exact linear phase and multi rate operation. Digital filtering can be applied to very low frequency signals, such as those occurring in biomedical and seismic applications very efficiently. In addition, the characteristics of digital filters can be changed or adapted by simply changing the content of a finite number of registers, thus multiple filtering tasks can be performed by one programmable digital filter without the need to replicate the hardware. With the ever increasing number of applications involving digital filters, the variety of requirements that have to be met by digital filters has increased. As a result, design techniques that are capable of satisfying the required design requirements are becoming an important necessity.

2. Background:

2.1 Finite Impulse Response Filters

FIR filters are digital filters with finite impulse response. They are also known as non-recursive digital filters as they do not have the feedback even though recursive algorithms can be used for FIR filter realization. FIR filters can be designed using different methods, but most of them are based on ideal filter approximation. The objective is not to achieve ideal characteristics, but to achieve sufficiently good characteristics of a filter. The transfer function of FIR filter approaches the ideal as the filter order increases, thus increasing the complexity and amount of time needed for processing input samples of a signal being filtered. In order that the phase characteristic of a FIR filter is linear, the impulse response must be symmetric or anti-symmetric, which is expressed in the following way.
h[n] = h[N-n-1], symmetric impulse response

h[n] = -h[N-n-1], anti-symmetric impulse response

One of the drawbacks of FIR filters is a high order of designed filter. The order of FIR filter is remarkably higher as compared to an IIR filter with the same frequency response. This is the reason why it is so important to use FIR filters only when the linear phase characteristic is very important. A number of delay lines contained in a filter, i.e. a number of input samples that should be saved for the purpose of computing the output sample, determine the order of a filter. For example, if the filter is assumed to be of order 10, it means that it is necessary to save 10 input samples preceding the current sample. All eleven samples will affect the output sample of FIR filter. The transform function of a typical FIR filter can be expressed as a polynomial of a complex variable z⁻¹. All the poles of the transfer function are located at the origin. For this reason, FIR filters are guaranteed to be stable, whereas IIR filters have potential to become unstable. Most FIR filter design methods are based on ideal filter approximation. The resulting filter approximates the ideal characteristic as the filter order increases, thus making the filter and its implementation more complex. The filter design process starts with specifications and requirements of the desirable FIR filter. The method to be used in the filter design process depends on the filter specifications and implementation. Each of the given methods has its advantages and disadvantages. Thus, it is very important to carefully choose the right method for FIR filter design. Due to its simplicity and efficiency, the window method is most commonly used method for designing filters. The sampling frequency method is easy to use, but filters designed this way have small attenuation in the stop band.

The filter design process can be described as an optimization problem where each requirement contributes with a term to an error function which should be minimized. Filter design is the process of designing a signal processing filter that satisfies a set of requirements, some of which are contradictory. The purpose is to find a realization of the filter that meets each of the requirements to a sufficient degree to make it useful. Consequently, there is a need for optimization-based methods that can be used to design digital filters that would satisfy prescribed specifications. However, optimization problems for the design of digital filters are often complex, highly nonlinear, and multimodal in nature. In my thesis, I am trying to find an optimum filter design solution on the minimization of filter coefficient error while maintaining higher efficiency. This thesis work is carried out to find an optimal digital filter structure using Parks-McClellan algorithm and Genetic algorithm as optimization techniques.

- A high-shelf filter passes all frequencies, but increases or reduces frequencies above the shelf frequency by specified amount.
- A peak EQ makes a peak or a dip in the frequency response, commonly used in parametric equalizers.

An important parameter is the required frequency response. In particular, the steepness and complexity of the response curve is a deciding factor for the filter order and feasibility. A first order recursive filter will only have a single frequency-dependent component. This means that the slope of the frequency response is limited to 6 dB per octave. For many purposes, this is not sufficient. To achieve steeper slopes, higher order filters are required. In relation to the desired frequency function, there may also be an accompanying weighting function which describes, for each frequency, how important it is that the resulting frequency function approximates the desired one. The larger weight, the more important is a close approximation.

### 2.2 FIR Filter Design Specifications

A filter with linear-phase response is desirable in many applications, notably image processing and data transmission. One of the desirable characteristics of FIR filters is that they can be designed very easily to have linear phase. In the phase response of the FIR filters, both the pass band and stop band ripples and the transition width are undesirable but unavoidable deviations from the response of an ideal low pass filter when approximating with a finite impulse response. Practical FIR designs typically consist of filters that meet certain design specifications i.e. those have a transition width and maximum pass band/stop band ripples that do not exceed allowable values. In addition, one must select the filter order, or equivalently, the length of the truncated impulse response. The transition of the frequency response from pass band to stop band defines the transition band or transition region of the filter. The pass band frequency (ωp) defines the edge of the pass band. The stop band frequency (ωs) defines the beginning of the stop band. The width of the transition band is given by (ωs - ωp). In many applications the specifications are given in terms of absolute frequency in Hertz rather than in terms of normalized frequency. Conversion between one and the other is straightforward. The normalized frequency is related to absolute frequency by

\[
\frac{\omega}{\omega_s}
\]

Where f is absolute frequency in cycles/second, fs is the sampling frequency in samples/second, and ω is normalized frequency in radians/sample. FIR filters are designed with the specifications such that the transition width, pass band and stop band ripples do not exceed the allowable values. An impulse consists of cosine function of all frequencies of equal magnitude with phase equal to zero at the time position of the impulse i.e. each cosine wave of different frequency has a peak at the position of the impulse. This position is called as Zero Phase Reference Point. Thus if an impulse is fed into a filter, its impulse response will fully define the characteristics of the filter in the time domain. Computing the FFT of the filter impulse response will provide the phase and magnitude response of the filter in the frequency domain. An FIR filter is obtained by specifying its coefficients. The coefficients are the samples of the filters impulse response. Generally the filter characteristics are specified in the frequency domain and the impulse response is obtained by computing the IFFT. The impulse response is then sampled to obtain the filter coefficients at the desired sample rate.

The figure below shows the typical deviations from the ideal low pass filter when approximating with an FIR filter.
3. Proposed Algorithm:

3.1 Design of Fractional Delay Filters

Variable fractional-delay (VFD) filters belong to a branch of variable digital filters which are applied in applications in which the frequency characteristics need to be adjustable online without redesigning a new filter. Due to their wide applications in signal processing and communication systems, the design of VFD filters have received considerable attention. Fractional-delay (FD) filters are a type of digital filter designed for band limited interpolation. Band limited interpolation is a technique for evaluating a signal sample at an arbitrary point in time, even if it is located somewhere between two sampling points. The value of the sample obtained is exact because the signal is band limited to half the sampling rate (Fs/2). This implies that the continuous-time signal can be exactly regenerated from the sampled data. Ideally, an FD filter is required to have a constant amplitude response of unity and a phase response that is linear with respect to some prescribed pass band say.

\[ 0 < \omega_p \leq 90^\circ \]

Where \( \omega_p \) is the pass band edge. Furthermore, the fractional delay realized should be adjustable without changing the filter coefficients. Digital fractional delay (FD) filters are useful tools to fine-tune the sampling instants of signals. They are typically found in the synchronization of digital modems where the delay parameter varies over time. The major steps to be followed in the execution of this work:-

**Step1:** Variable Fractional delay FIR Filter is designed using the Farrow Structure, a popular method for implementing time-varying FIR FD filters. To compute the output of a fractional delay filter, we need to estimate the values of the input signal between the existing discrete-time samples. Special interpolation filters can be used to compute new sample values at arbitrary points. Among those, polynomial-based filters are of particular interest because a special structure - the Farrow structure, permits simple handling of coefficients. In particular, the tunability of the Farrow structure makes it well-suited for practical hardware implementations. To design a filter means to select the coefficients such that the system has specific characteristics. The required characteristics are stated in filter specifications. Most of the time filter specifications refer to the frequency response of the filter. The advantage of the Farrow structure over a Direct-Form FIR resides in its tunability. In many practical applications, the delay is time-varying. For each new delay we would need a new set of coefficients in the Direct-Form implementation but with a Farrow implementation, the polynomial coefficients remain constant. In this design each of the impulse response coefficients are modeled as \( M^k \) order polynomials of the delay variable which implemented the variable filter as a linear combination of \( M+1 \) filters as shown below.

![Fig 2. Farrow structure with fixed sub filters \( S_i(z) \) and a variable FD of \( \mu \) [3]](image)

The Farrow Structure shown in fig.2 is composed of fixed linear phase finite length impulse response (FIR) subfilters \( S_i(z) \), \( k=0,1, \ldots, L \) of order \( N_k \) as well as the variable multipliers \( \mu \). The transfer function is

\[
H(z, \mu) = \sum_{k=0}^{L} S_k(z) \mu^k, |\mu| \leq 0.5 \quad \ldots \quad 3.1
\]

The design parameters are hence the number of subfilters, \( L+1 \), the order of the sub filters \( N_k \) and the coefficients of the sub filters \( S_k(z) \). The overall structure can approximate FD filters with an adjustable \( \mu \) over a frequency range of \( \omega \in [0,\omega_p] \). In the transfer function of the Farrow Structure, different sub filters are weighted by different powers of the FD value. As both the FD value and its powers are smaller than 0.5, these are used as diminishing weighting functions. The approximation error for each sub filter is then increased in proportion to the powers of the FD value. A new distribution to the orders of the Farrow sub filters is given. These diminishing weighting functions are then used in the filter design so as to obtain the optimum values iteratively. Sub filters of even order are considered.

**Step2:** Parks-McClellan method (also known as the Equiripple, Optimal, or Mini max method) with the Remez exchange algorithm is used to find an optimal equiripple set of coefficients to design an optimal linear phase filter. This is a standard method for the FIR filter design which minimizes the filter length for a particular set of design constraints. This method is used to design
linear phase, symmetric or anti symmetric filters of any standard type. Better filters result from minimization of maximum error in both, the stop band and the pass band of the filter which leads to equiripple filters. Such filters are an optimum approximate and can be achieved using algorithmic techniques. In this algorithm to design FIR filters, some of its parameters such as the filter length $(M)$, pass band and stop band normalized frequencies $(\omega_p, \omega_s)$, maximum of the absolute ripple in the pass band and stop band $(d_p, d_s)$ are fixed and the remaining parameters are to be optimized. Parameters $M$, $d_p$, and $d_s$ are fixed while the remaining parameters are optimized. The Parks–McClellan (PM) algorithm is the most popular approach for optimum FIR filters design due to its flexibility and computational efficiency. In the PM algorithm, an approximate error function is defined by

$$E(\omega) = \| G(\omega) \| H_j(\omega) - H(\omega) \|$$

Where $H_j(\omega)$ and $H(\omega)$ are the frequency responses of the desired and the approximate filters respectively. $G(\omega)$ is the weighting function. It is used to provide weighting of the approximation error differently in different frequency bands. The filter is optimized in the sense that the maximum weighted error is minimized. However this algorithm does not allow explicit selection of the maximum of the absolute ripple in the pass band and stop band, one can only specify their ratio. Furthermore, the PM gives floating point coefficients which require quantization. Here the user specifies a desired frequency response, a weighting function for errors from this response, and a filter order $N$. The algorithm then finds the set of $N+1$ coefficients that minimize the maximum deviation from the ideal. This finds the filter that is as close to the desired response. This method is particularly easy in practice since it includes a program that takes the desired filter and $N$, and returns the optimum coefficients. The resulting filters minimize the maximum error between the desired frequency response and the actual frequency response by spreading the approximation error uniformly over each band. Such filters that exhibit equiripple behavior in both the pass band and the stop band, and are sometimes called equiripple filters.

The computational effort is linearly proportional to the length of the filter. In Matlab this method is available as remez(). Use the (remezord) command to estimate the order of the optimal P-Mc FIR filter.

Step3: Generalized optimization techniques are then used to minimize (or maximize) a given function, known as the objective function, or cost function. A linear Optimization problem is the one whose objective function is a linear function of the input. Optimization algorithms generally take a starting guess point and change the variables subjected to the constraints in such a way that it decrease (or increase) the objective function. Some sort of termination condition is then required.

Step4: The two Optimization approaches for the design of fractional delay filters based on a Parks-McClellan Algorithm and GA are compared. In the first approach, the coefficients of an FD FIR Filters are determined based on the Farrow structure. In the second approach, the FD filter is designed by using the all pass FIR-based Farrow structure. In both approaches, the designs obtained are free of quantization errors. Simulation results for both approaches were compared.

4. Results and Discussions:

In this chapter, the techniques Parks–McClellan (PM) algorithm and Genetic algorithms (GA) are used to design optimum FIR filters for various cases. In all the cases, the filter to be designed is assumed to be a linear phase LPF with even length. Phase linearity of the filter is guaranteed by assuming symmetry of the approximate filter which reduces the dimensions of the optimization problem. The design of digital filters basically means finding the values of filter coefficients so that given filter specification are achieved. The Design Specifications of the filter under consideration are discussed below with different parameters used in the design process.

Case 1: (Low Pass filter of order 10)

To design a minimum-order low pass filter with a 10 MHz pass band cutoff frequency and 20 MHz stop band cutoff frequency, with a sampling frequency of 50 MHz, at least 30 dB attenuation in the stop band, and less than 3 dB of ripple in the pass band using the PM algorithm.

4.1 Calculation of Filter coefficients:

The design of digital filter means basically finding the values of filter coefficients so that given filter specification are achieved. PM algorithm is used to design a minimum order filter and to obtain the filter coefficients. The minimum order calculated through PM algorithm is equal to 10 as the total number of filter coefficients are taken to be 11 from a0 to a11. The PM gives floating point coefficients which require quantization. This motivates the use of Genetic algorithm which gives quantized filter coefficients. The following Table compares the values of the three filter coefficients obtained by Ideal Filtering, through Fractional Delay and PM algorithm and finally after the filter optimization using Genetic Algorithm.
Filter Coefficients | Ideal filtering | Through fractional delay and PM Algorithm | After optimization through GA Algorithm
---|---|---|---
a0 | 0.5006 | -0.195 | -1.2548
\(a_1\) | 0.0038 | 0.231 | 0.9254
\(a_2\) | 0.0002 | 0.1327 | -1.2522
\(a_3\) | 0.0051 | 0.2677 | 1.1258
\(a_4\) | 0.0010 | 0.3298 | -1.1754
\(a_5\) | -0.0013 | 0.2677 | 1.2847
\(a_6\) | 0.0029 | 0.1327 | 0.1658
\(a_7\) | -0.0001 | 0.231 | 1.2145
\(a_8\) | 0.0303 | -0.195 | -0.9957
\(a_9\) | -0.0007 | -0.270 | 1.1252
\(a_{10}\) | -0.0024 | -0.194 | -1.0892

Table: calculated filter coefficients values through PM and GA algorithms

5. Conclusions and Future Scope:

In this paper, a new method for the minimization of the root-mean-square error of variable group-delay response has been proposed for the design of VFD FIR digital filters. To overcome the nonlinear optimization for minimization, the proposed iterative method can be successfully used, and the experimental results may be show that the performance in group-delay response and the convergence of the iterative method are satisfactory.

References:


