A Study On Intuitionistic Anti L-Fuzzy Normal M-Subgroups

P. Pandiammal

Associate Professor, Department of Mathematics,
PSNA College of Engg., & Tech., Dindigul, Tamil Nadu, India

Abstract – In this paper, we introduce the concept of intuitionistic anti L-fuzzy normal M-subgroups and investigate some related properties.

Keywords: Intuitionistic fuzzy subsets; Intuitionistic anti fuzzy subgroups; Intuitionistic anti L-fuzzy M-subgroups; Intuitionistic anti L-fuzzy normal M-subgroups; M-homomorphism.

I. INTRODUCTION

A fuzzy set theory has developed in many directions and finding application in a wide variety of fields. Zadeh's classical paper [20] of 1965 introduced the concepts of fuzzy sets and fuzzy set operations. The study of fuzzy groups was started by Rosenfeld [16] and it was extended by Roventa [17] who have introduced the concept of fuzzy groups operating on fuzzy sets and many researchers [1,7,9,10] are engaged in extending the concepts. The concept of intuitionistic fuzzy set was introduced by Atanassov, K.T [2,3], as a generalization of the notion of fuzzy sets. Choudhury, F.P et al [6] defined a fuzzy subgroup and fuzzy homomorphism. Palaniappan, N and Muthuraj, [11] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. Pandianmal, P, Natarajan, R and Palaniappan, N, [13] defined the homomorphism, anti-homomorphism of an anti L-fuzzy M-subgroup. Pandiammal, P,[14] defined the homomorphism, anti-homomorphism of an intuitionistic anti L-fuzzy M-subgroups. In this paper we introduce and discuss the algebraic nature of intuitionistic anti L-fuzzy normal M-subgroups with operator and obtain some related results.

II. PRELIMINARIES

2.1 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be anti L-fuzzy M-subgroup (ALFMSG) of G if its satisfies the following axioms:

(i) \( \mu_A( mxy ) \leq \mu_A(x) \lor \mu_A(y) \),

(ii) \( \mu_A(x^{-1}) \leq \mu_A(x) \), for all \( x \) and \( y \) in G.

2.2 Definition: Let \( (G, \cdot) \) be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an intuitionistic L-fuzzy M-subgroup (ILFMSG) of G if the following conditions are satisfied:

(i) \( \mu_A( mxy ) \geq \mu_A(x) \land \mu_A(y) \),

(ii) \( \mu_A(x^{-1}) \geq \mu_A(x) \),

(iii) \( \nu_A( mxy ) \leq \nu_A(x) \lor \nu_A(y) \),

(iv) \( \nu_A(x^{-1}) \leq \nu_A(x) \), for all \( x \) & \( y \) in G.

2.3 Definition: Let \( (G, \cdot) \) and \( (G', \cdot) \) be any two M-groups. Let \( f : G \rightarrow G' \) be any function and A be an intuitionistic L-fuzzy M-subgroup in G, V be an intuitionistic L-fuzzy M-subgroup in f \( (G) = G' \), defined by \( \mu_V(y) = \text{SUP} \mu_A(x) \) and \( \nu_V(y) = \inf_{x \in f^{-1}(y)} \nu_A(x) \), for all \( x \) in G and \( y \) in G'.

Then A is called a preimage of V under f and is denoted by \( f^{-1}(V) \).
2.4 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB = \{ (x, y) : \mu_{AxB}(x, y), v_{AxB}(x, y) \} / for all x in G and y in H \}, where 
\mu_{AxB}(x, y) = \mu_A(x) \wedge \mu_B(y) and 
v_{AxB}(x, y) = v_A(x) \vee v_B(y).

2.5 Definition: Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be conjugate intuitionistic L-fuzzy M-subgroups of G if for some g in G, \mu_A(x) = \mu_B(g^{-1}xg) and v_A(x) = v_B(g^{-1}xg), for every x in G.

2.6 Definition: Let A be an intuitionistic L-fuzzy subset in a set S, the strongest intuitionistic L-fuzzy relation on S, that is an intuitionistic L-fuzzy relation on A is V given by 
\mu_V(x, y) = \mu_A(x) \wedge \mu_B(y) and 
v_V(x, y) = v_A(x) \vee v_B(y), for all x and y in S.

III. INTUITIONISTIC ANTI L-FUZZY NORMAL M-SUBGROUPS

3.1 Definition: An intuitionistic fuzzy subset \mu in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following axioms are satisfied.

(i) \( \mu_A(xy) \leq \mu_A(x) \vee \mu_A(y) \),
(ii) \( \mu_A(x^{-1}) \leq \mu_A(x) \),
(iii) \( v_A(xy) \geq v_A(x) \wedge v_A(y) \),
(iv) \( v_A(x^{-1}) \geq v_A(x) \), for all x and y in G.

3.2 Proposition: Let G be a group. An intuitionistic fuzzy subset \mu in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following conditions are satisfied.

(i) \( \mu_A(xy) \leq \mu_A(x) \vee \mu_A(y) \),
(ii) \( v_A(xy) \geq v_A(x) \wedge v_A(y) \), for all x and y in G.

3.3 Definition: Let G be an M-group and \mu be an intuitionistic anti fuzzy group of G. If 
\( \mu_A(mx) \leq \mu_A(x) \) and \( v_A(mx) \geq v_A(x) \) for all x in G and m in M then \mu is said to be an intuitionistic anti fuzzy subgroup with operator of G. We use the phrase \mu is an intuitionistic anti L-fuzzy M-subgroup of G.

3.4 Example: Let H be M-subgroup of an M-group G and let \( A = (\mu_A, v_A) \) be an intuitionistic fuzzy set in G defined by

\[ \mu_A(x) = \begin{cases} 
0.3 & ; x \in H \\
0.5 & ; \text{otherwise} 
\end{cases} \]

\[ v_A(x) = \begin{cases} 
0.6 & ; x \in H \\
0.3 & ; \text{otherwise} 
\end{cases} \]

For all x in G. Then it is easy to verify that \( A = (\mu_A, v_A) \) is an anti fuzzy M-subgroup of G.

3.5 Definition: Let A and B be any two intuitionistic anti L-fuzzy M-subgroups of a M-group (G, ·). Then A and B are said to be conjugate intuitionistic anti L-fuzzy normal M-subgroups of G if for some g in G, 
\( \mu_A(x) = \mu_B(g^{-1}xg) \) and \( v_A(x) = v_B(g^{-1}xg) \), for every x in G.

3.6 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB = \{ (x, y), \mu_{AxB}(x, y), v_{AxB}(x, y) \} / for all x in G and y in H \}, where 
\mu_{AxB}(x, y) = \mu_A(x) \vee \mu_B(y) and 
v_{AxB}(x, y) = v_A(x) \wedge v_B(y).
3.7 **Definition:** Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group ( G, · ). Then A and B are said to be conjugate intuitionistic L-fuzzy M-subgroups of G if for some g in G, \( \mu_A(x) = \mu_B(g^{-1}xg) \) and \( \nu_A(x) = \nu_B(g^{-1}xg) \), for every x in G.

3.8 **Definition:** Let ( G, · ) be a M-group. An intuitionistic L-fuzzy M-subgroup A of G is said to be an **intuitionistic Anti L-fuzzy normal M-subgroup (IALFNMMSG)** of G if the following conditions are satisfied:

(i) \( \mu_A(xy) = \mu_A(yx) \),

(ii) \( \nu_A(xy) = \nu_A(yx) \), for all x and y in G.

3.9 **Definition:** An intuitionistic L-fuzzy subset A of a set X is said to be **normalized** if there exist x in X such that \( \mu_A(x) = 1 \) and \( \nu_A(x) = 0 \).

3.10 **Definition:** Let ( G, · ) be a M-group. An intuitionistic L-fuzzy M-subgroup A of G is said to be an **intuitionistic L-fuzzy characteristic M-subgroup (ILFCMSG)** of G if the following conditions are satisfied:

(i) \( \mu_A(x) = \mu_A(f(x)) \),

(ii) \( \nu_A(x) = \nu_A(f(x)) \), for all x in G and f in AutG.

3.3 **Theorem:** If A and B are intuitionistic anti L-fuzzy M-subgroups of the M-groups G and H, respectively, then AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH.

3.4 **Theorem:** Let an intuitionistic anti L-fuzzy M-subgroup A of a M-group G be conjugate to an intuitionistic anti L-fuzzy M-subgroup M of G and an intuitionistic anti L-fuzzy M-subgroup B of a M-group H be conjugate to an intuitionistic anti L-fuzzy M-subgroup N of H. Then an intuitionistic anti L-fuzzy M-subgroup AxB of a M-group GxH is conjugate to an intuitionistic anti L-fuzzy M-subgroup MxN of GxH.

3.5 **Theorem:** Let A and B be intuitionistic anti L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e' are the identity elements of G and H, respectively. If AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH, then at least one of the following two statements must hold.

(i) \( \mu_B(e') \leq \mu_A(x) \) and \( \nu_B(e') \geq \nu_A(x) \), for all x in G,

(ii) \( \mu_A(e) \leq \mu_B(y) \) and \( \nu_A(e) \geq \nu_B(y) \), for all y in H.

3.6 **Theorem:** Let A and B be intuitionistic anti L-fuzzy subsets of the M-groups G and H, respectively and AxB is an intuitionistic anti L-fuzzy M-subgroup of GxH. Then the following are true:

(i) if \( \mu_A(x) \leq \mu_B(e') \) and \( \nu_A(x) \geq \nu_B(e') \), then A is an intuitionistic anti L-fuzzy M-subgroup of G.

(ii) if \( \mu_B(x) \leq \mu_A(e) \) and \( \nu_B(x) \geq \nu_A(e) \), then B is an intuitionistic anti L-fuzzy M-subgroup of H.

(iii) either A is an intuitionistic anti L-fuzzy M-subgroup of G or B is an intuitionistic anti L-fuzzy M-subgroup of H, where e and e' are the identity elements of G and H, respect.,

3.7 **Theorem:** Let ( G, · ) and ( G', · ) be any two M-groups. The homomorphic image of an
intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G'.

3.8 Theorem: Let (G, ⋅) and (G₁, ⋅) be any two M-groups. The homomorphic pre-image of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G.

3.9 Theorem: Let (G, ⋅) and (G₁, ⋅) be any two M-groups. The anti-homomorphic image of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G'.

3.10 Theorem: Let (G, ⋅) and (G₁, ⋅) be any two M-groups. The anti-homomorphic pre-image of an intuitionistic anti L-fuzzy M-subgroup of G is an intuitionistic anti L-fuzzy M-subgroup of G.

3.11 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H and f is an isomorphism from a M-group G onto H. Then A ⋅ f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

3.12 Theorem: Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H and f is an anti-isomorphism from a M-group G onto H. Then A ⋅ f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

3.13 Theorem: Let (G, ⋅) be a M-group. If A and B are two intuitionistic anti L-fuzzy normal M-subgroups of G, then their intersection A ⋈ B is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Proof: Let x and x in G and A = {〈x, μₐ(x), νₐ(x)〉/x ∈ G} and B = {〈x, μₐ(x), νₐ(x)〉/x ∈ G} be an intuitionistic anti L-fuzzy normal M-subgroups of G.

Let C = A ⋈ B and C = {〈x, μₐ(x), νₐ(x)〉/x ∈ G}.

Then, Clearly C is an intuitionistic anti L-fuzzy M-subgroup of a M - group G, since A and B are two intuitionistic anti L-fuzzy M-subgroups of a M-group G.

Now, μₐ(xy) = μₐ(xy) V μₐ(xy) = μₐ(xy) V μₐ(xy) = μₐ(xy).

Therefore, μₐ(xy) = μₐ(xy), for all x and y in G.

And, νₐ(xy) = νₐ(xy) A νₐ(xy) = νₐ(xy) A νₐ(xy) = νₐ(xy).

Therefore, νₐ(xy) = νₐ(xy) , for all x and y in G.

Hence A ⋈ B is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

3.14 Theorem: Let (G, ⋅) be a M-group. The intersection of a family of intuitionistic anti L-fuzzy normal M-subgroups of G is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Proof: Let {Aᵢ}₁ ≤ i ≤ n be a family of intuitionistic anti L-fuzzy normal M-subgroups of a M-group G and A = ∩₁ ≤ i ≤ n Aᵢ. Then for x and y in G, clearly the intersection of a family of intuitionistic anti L-fuzzy M-subgroups of a M-group G is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

Now, μₐ(xy) = sup₁ ≤ i ≤ n μₐ(xy) = sup₁ ≤ i ≤ n μₐ(xy) = μₐ(xy).

Therefore, μₐ(xy) = μₐ(xy), for all x and y in G.
And, \( v_A(xy) = \inf_{i \in I} v_A(xy) \)
\[ = \inf_{i \in I} v_A(yx) \]
\[ = v_A(yx). \]
Therefore, \( v_A(xy) = v_A(yx) \), for all \( x \) and \( y \) in \( G \).

Hence the intersection of a family of intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \) is an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \).

**3.15 Theorem:** If \( A \) is an intuitionistic anti \( L \)-fuzzy characteristic \( M \)-subgroup of a \( M \)-group \( G \), then \( A \) is an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \).

**Proof:** Let \( A \) be an intuitionistic anti \( L \)-fuzzy characteristic \( M \)-subgroup of a \( M \)-group \( G \), \( x \) and \( y \) in \( G \).

Consider the map \( f : G \to G \) defined by \( f(x) = yxy^{-1} \).

Clearly, \( f \) in \( \text{Aut}G \).

Now, \( \mu_A(xy) = \mu_A(f(xy)) \)
\[ = \mu_A(y(xy)y^{-1}) = \mu_A(yx). \]
Therefore, \( \mu_A(xy) = \mu_A(yx) \), for all \( x \) and \( y \) in \( G \).

Again, \( \nu_A(xy) = \nu_A(f(xy)) \)
\[ = \nu_A(y(xy)y^{-1}) \]
\[ = \nu_A(yx). \]
Therefore, \( \nu_A(xy) = \nu_A(yx) \), for all \( x \) and \( y \) in \( G \).

Hence \( A \) is an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \).

**3.16 Theorem:** An intuitionistic anti \( L \)-fuzzy \( M \)-subgroup \( A \) of a \( M \)-group \( G \) is an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of \( G \) if and only if \( A \) is constant on the conjugate classes of \( G \).

**Proof:** Suppose that \( A \) is an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \). Let \( x \) and \( y \) in \( G \).

Now, \( \mu_A(y^{-1}xy) = \mu_A(xy^{-1}) = \mu_A(x) \).

Therefore, \( \mu_A(y^{-1}xy) = \mu_A(x) \), for all \( x \) and \( y \) in \( G \). And, \( v_A(y^{-1}xy) = v_A(xy^{-1}) = v_A(x) \).

Therefore, \( v_A(y^{-1}xy) = v_A(x) \), for all \( x \) and \( y \) in \( G \). Hence \( (x) = \{ y^{-1}xy \mid y \in G \} \).

Hence \( A \) is constant on the conjugate classes of \( G \).
Conversely, suppose that \( A \) is constant on the conjugate classes of \( G \).

Then, \( \mu_A(xy) = \mu_A(xyxx^{-1}) \)
\[ = \mu_A((xy)x^{-1}) \]
\[ = \mu_A(yx). \]
Therefore, \( \mu_A(xy) = \mu_A(yx) \), for all \( x \) and \( y \) in \( G \).

And, \( v_A(xy) = v_A(xyx^{-1}) \)
\[ = v_A((yx)x^{-1}) \]
\[ = v_A(yx). \]
Therefore, \( v_A(xy) = v_A(yx) \), for all \( x \) and \( y \) in \( G \).

Hence \( A \) is an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \).

**3.17 Theorem:** Let \( A \) be an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \). Then for any \( y \) in \( G \), we have \( \mu_A(yxy^{-1}) = \mu_A(y^{-1}xy) \) and \( \nu_A(yxy^{-1}) = \nu_A(y^{-1}xy) \), for every \( x \) in \( G \).

**Proof:** Let \( A \) be an intuitionistic anti \( L \)-fuzzy normal \( M \)-subgroup of a \( M \)-group \( G \).
For any \( y \) in \( G \), we have, 
\[
\mu_A(yxy^{-1}) = \mu_A(yy^{-1}x)
\]
\[
= \mu_A(x)
\]
\[
= \mu_A(xyy^{-1}) = \mu_A(y^{-1}xy).
\]

Therefore, \( \mu_A(yxy^{-1}) = \mu_A(y^{-1}xy) \), for all \( x \) & \( y \) in \( G \).

And, 
\[
\nu_A(yxy^{-1}) = \nu_A(yy^{-1}x)
\]
\[
= \nu_A(x)
\]
\[
= \nu_A(xyy^{-1}) = \nu_A(y^{-1}xy).
\]

Therefore, \( \nu_A(yxy^{-1}) = \nu_A(y^{-1}xy) \), for all \( x \) & \( y \) in \( G \).

3.18 Theorem: An intuitionistic anti L-fuzzy M-subgroup \( A \) of a M-group \( G \) is normalized if and only if \( \mu_A(e) = 1 \) and \( \nu_A(e) = 0 \), where \( e \) is the identity element of the M-group \( G \).

**Proof:** If \( A \) is normalized, then there exists \( x \) in \( G \) such that \( \mu_A(x) = 1 \) and \( \nu_A(x) = 0 \), but by properties of an intuitionistic anti L-fuzzy M-subgroup \( A \) of the M-group \( G \), \( \mu_A(x) \leq \mu_A(e) \) and \( \nu_A(x) \geq \nu_A(e) \), for every \( x \) in \( G \).

Since \( \mu_A(x) = 1 \) and \( \nu_A(x) = 0 \) and \( \mu_A(x) \leq \mu_A(e) \) and \( \nu_A(x) \geq \nu_A(e) \), \( 1 \leq \mu_A(e) \) and \( 0 \geq \nu_A(e) \).

But, \( \nu_A(e) \). Hence \( \mu_A(e) = 1 \) and \( \nu_A(e) = 0 \).

Conversely, if \( \mu_A(e) = 1 \) and \( \nu_A(e) = 0 \), then by the definition of normalized intuitionistic anti L-fuzzy subset, \( A \) is normalized.

3.19 Theorem: Let \( A \) and \( B \) be intuitionistic anti L-fuzzy M-subgroups of the M-groups \( G \) and \( H \), respectively. If \( A \) and \( B \) are intuitionistic anti L-fuzzy normal M-subgroups, then \( AxB \) is an intuitionistic anti L-fuzzy normal M-subgroup of \( GxH \).

**Proof:** Let \( A \) and \( B \) be intuitionistic anti L-fuzzy normal M-subgroups of the M-groups \( G \) and \( H \), respectively.

Clearly \( AxB \) is an intuitionistic anti L-fuzzy M-subgroup of \( GxH \).

Let \( x_1 \) and \( x_2 \) be in \( G \), \( y_1 \) and \( y_2 \) be in \( H \). Then \( (x_1, y_1) \) and \( (x_2, y_2) \) are in \( GxH \).

Now, 
\[
\mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}(x_1x_2, y_1y_2)
\]
\[
= \mu_{AxB}(x_1x_2) \land \mu_{B}(y_1y_2)
\]
\[
= \mu_{A}(x_2x_1) \land \mu_{B}(y_2y_1)
\]
\[
= \mu_{AxB}(x_2x_1, y_2y_1)
\]
\[
= \mu_{AxB}[(x_2, y_2)(x_1, y_1)].
\]

Therefore, \( \mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}[(x_2, y_2)(x_1, y_1)] \), for all \( x_1, x_2 \) in \( G \) and \( y_1 \) and \( y_2 \) in \( H \).

And, 
\[
\nu_{AxB}[(x_1, y_1)(x_2, y_2)]
\]
\[
= \nu_{AxB}(x_1x_2, y_1y_2)
\]
\[
= \nu_{A}(x_1x_2) \lor \nu_{B}(y_1y_2)
\]
\[
= \nu_{A}(x_2x_1) \lor \nu_{B}(y_2y_1)
\]
\[
= \nu_{AxB}(x_2x_1, y_2y_1)
\]
\[
= \nu_{AxB}[(x_2, y_2)(x_1, y_1)].
\]

Therefore, \( \nu_{AxB}[(x_1, y_1)(x_2, y_2)] = \nu_{AxB}[(x_2, y_2)(x_1, y_1)] \), for all \( x_1, x_2 \) in \( G \) and \( y_1 \) and \( y_2 \) in \( H \).

Hence \( AxB \) is an intuitionistic anti L-fuzzy normal M-subgroup of \( GxH \).

3.20 Theorem: Let an intuitionistic anti L-fuzzy normal M-subgroup \( A \) of a M-group \( G \) be conjugate to an intuitionistic anti L-fuzzy normal M-subgroup \( M \) of \( G \) and an intuitionistic anti L-fuzzy normal M-subgroup \( B \) of a M-group \( H \) be conjugate to an intuitionistic anti L-fuzzy normal M-subgroup \( N \) of \( H \). Then an intuitionistic anti L-fuzzy normal M-subgroup \( AxB \) of a M-group \( GxH \) is conjugate to an intuitionistic anti L-fuzzy normal M-subgroup \( MxN \) of \( GxH \).
Proof: It is trivial.

3.21 Theorem: Let $A$ and $B$ be intuitionistic anti L-fuzzy subsets of the M-groups $G$ and $H$, respectively. Suppose that $e$ and $e'$ are the identity element of $G$ and $H$, respectively. If $A \times B$ is an intuitionistic anti L-fuzzy normal M-subgroup of $G \times H$, then at least one of the following two statements must hold.

(i) $\mu_B(e') \leq \mu_A(x)$ and $\nu_B(e') \geq \nu_A(x)$, for all $x$ in $G$,
(ii) $\mu_A(e) \leq \mu_B(y)$ and $\nu_A(e) \geq \nu_B(y)$, for all $y$ in $H$.

Proof: It is trivial.

3.22 Theorem: Let $A$ and $B$ be intuitionistic anti L-fuzzy subsets of the M-groups $G$ and $H$, respectively and $A \times B$ is an intuitionistic anti L-fuzzy normal M-subgroup of $G \times H$. Then the following are true:

(i) if $\mu_A(x) \geq \mu_B(e')$ and $\nu_A(x) \leq \nu_B(e')$, then $A$ is an intuitionistic L-fuzzy normal M-subgroup of $G$.
(ii) if $\mu_B(x) \geq \mu_A(e)$ and $\nu_B(x) \leq \nu_A(e)$, then $B$ is an intuitionistic anti L-fuzzy normal M-subgroup of $H$.
(iii) either $A$ is an intuitionistic anti L-fuzzy normal M-subgroup of $G$ or $B$ is an intuitionistic anti L-fuzzy normal M-subgroup of $H$.

Proof: It is trivial.

4. INTUITIONISTIC ANTI L-FUZZY NORMAL M-SUBGROUPS OF M-GROUPS UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM

4.1 Theorem: Let $(G, \cdot)$ and $(G', \cdot)$ be any two M-groups. The homomorphic image of an intuitionistic anti L-fuzzy normal M-subgroup of $G$ is an intuitionistic anti L-fuzzy normal M-subgroup of $G'$.

Proof: Let $(G, \cdot)$ and $(G', \cdot)$ be any two M-groups and $f : G \rightarrow G'$ be a homomorphism.

That is $f(xy) = f(x)f(y)$, $f(mx) = mf(x)$, for all $x$ and $y$ in $G$ and $m$ in $M$.

Let $V = f(A)$, where $A$ is an intuitionistic anti L-fuzzy normal M-subgroup of $G$.

We have to prove that $V$ is an intuitionistic anti L-fuzzy normal M-subgroup of $G'$.

Now, for $f(x)$ and $f(y)$ in $G'$, we have clearly $V$ is an intuitionistic anti L-fuzzy M-subgroup of $G$.

Now, $\mu_V(f(x)f(y)) = \mu_V(f(xy))$
$\leq \mu_A(xy)$
$= \mu_A(xy)$
$\geq \mu_V(f(yx))$
$= \mu_V(f(y)f(x))$, which implies that
$\mu_V(f(x)f(y))$
$= \mu_V(f(y)f(x))$, for all $x$ and $y$ in $G$.

Now, $\nu_V(f(x)f(y)) = \nu_V(f(xy))$
$\geq \nu_A(xy)$
$= \nu_A(xy)$
$\leq \nu_V(f(yx))$
$= \nu_V(f(y)f(x))$, which implies that $\nu_V(f(x)f(y)) = \nu_V(f(y)f(x))$, for all $x$ and $y$ in $G$.

Hence $V$ is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group $G'$.

4.2 Theorem: Let $(G, \cdot)$ and $(G', \cdot)$ be any two M-groups. The homomorphic pre-image of an
intuitionistic anti L-fuzzy normal M-subgroup of G is an intuitionistic anti L-fuzzy normal M-subgroup of G. 

**Proof:** Let ( G, · ) and ( G¹, · ) be any two M-groups. Let f : G → G¹ be a homomorphism.

That is f(xy) = f(x)f(y), f(mx) = mf(x), for all x and y in G and m in M.

Let V=f(A), where V is an intuitionistic anti L-fuzzy normal M-subgroup of G¹.

We have to prove that A is an intuitionistic anti L-fuzzy normal M-subgroup of G.

Let x and y in G.

Then, clearly A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G, since V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G¹.

Now, \( \mu_A(xy) = \mu_V(f(xy)) \)
= \( \mu_V(f(x)f(y)) \)
= \( \mu_V(f(y)f(x)) \)
= \( \mu_A(yx) \),
which implies that \( \mu_A(xy) = \mu_A(yx) \), for all x and y in G.

Now, \( \nu_A(xy) = \nu_V(f(xy)) \)
= \( \nu_V(f(x)f(y)) \)
= \( \nu_V(f(y)f(x)) \)
= \( \nu_A(yx) \),
which implies that \( \nu_A(xy) = \nu_A(yx) \), for all x and y in G.

Hence A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

**4.3 Theorem:** Let ( G, · ) and ( G¹, · ) be any two M-groups. The anti-homomorphism image of an intuitionistic anti L-fuzzy normal M-subgroup of G is an intuitionistic anti L-fuzzy normal M-subgroup of G¹.

**Proof:** Let ( G, · ) and ( G¹, · ) be any two M-groups. Let f : G → G¹ be an anti-homomorphism.

That is f(xy) = f(y)f(x), f(mx) = mf(x), for all x and y in G and m in M.

Let V = f(A), where A is an intuitionistic anti L-fuzzy normal M-subgroup of G.

We have to prove that V is an intuitionistic anti L-fuzzy normal M-subgroup of G¹.

For f(x) and f(y) in G¹, clearly V is an intuitionistic anti L-fuzzy M-subgroup (IALFMSG) of a M-group G¹, since A is an intuitionistic anti L-fuzzy M-subgroup G.

Now, \( \mu_V(f(x)f(y)) = \mu_V(f(y)f(x)) \)
\( \leq \mu_A(yx) \)
\( \geq \mu_V(f(xy)) \)
which implies that 
\( \mu_V(f(x)f(y)) = \mu_V(f(y)f(x)) \), for all x and y in G.

And, \( \nu_V(f(x)f(y)) = \nu_V(f(y)f(x)) \)
\( \geq \nu_A(yx) \)
\( \leq \nu_V(f(xy)) \)
which implies that \( \nu_V(f(x)f(y)) = \nu_V(f(y)f(x)) \).

Hence V is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G¹.

**4.4 Theorem:** Let ( G, · ) and ( G¹, · ) be any two M-groups. The anti-homomorphism pre-image of an
intuitionistic anti L-fuzzy normal M-subgroup of G', is an intuitionistic anti L-fuzzy normal M-subgroup of G.

**Proof:** Let (G, ·) and (G', ·) be any two M-groups. Let f : G → G' be an anti-homomorphism.

That is f(xy) = f(y)f(x), f(mx) = mf(x), for all x and y in G and m in M. Let V = f(A), where V is an intuitionistic anti L-fuzzy normal M-subgroup of G'.

We have to prove that A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

Let x and y in G, we have clearly A is an intuitionistic anti L-fuzzy M-subgroup of a M-group G, since V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G'.

Now, µ_A(xy) = µ_V(f(xy))
= µ_V(f(y)f(x))
= µ_V(f(x)f(y))
= µ_V(f(yx))
= µ_A(xy), which implies that µ_A(xy) = µ_A(xy), for all x and y in G.

Now, ν_A(xy) = ν_V(f(xy))
= ν_V(f(y)f(x))
= ν_V(f(x)f(y))
= ν_V(f(yx))
= ν_A(xy), which implies that ν_A(xy) = ν_A(xy), for all x and y in G.

Hence A is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

**In the following Theorem ♦ is the composition operation of functions**

**4.5 Theorem:** Let A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H and f is an isomorphism from a M-group G onto H. If A is an intuitionistic anti L-fuzzy normal M-subgroup of H, then A◦f is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G.

**Proof:** Let x and y in G and A be an intuitionistic anti L-fuzzy M-subgroup of a M-group H.

We know that, A◦f is an intuitionistic anti L-fuzzy M-subgroup of a M-group G.

Then we have, (µ_A◦f)(xy) = µ_A( f(xy) )
= µ_A( f(x)f(y) )
= µ_A( f(y)f(x) )
= (µ_A◦f)(yx),
which implies that
(µ_A◦f)(xy)
= (µ_A◦f)(yx), for all x and y in G.

Now, (ν_A◦f)(xy) = ν_A( f(xy) )
= ν_A( f(x)f(y) )
= ν_A( f(y)f(x) )
= (ν_A◦f)(yx),
which implies that
(ν_A◦f)(xy)
= (ν_A◦f)(yx), for all x and y in G.
Hence $A^f$ is an intuitionistic anti $L$-fuzzy normal $M$-subgroup of a $M$-group $G$.

**4.6 Theorem:** Let $A$ be an intuitionistic anti $L$-fuzzy $M$-subgroup of a $M$-group $H$ and $f$ is an anti-isomorphism from a $M$-group $G$ onto $H$. If $A$ is an intuitionistic anti $L$-fuzzy normal $M$-subgroup of $H$, then $A^f$ is an intuitionistic anti $L$-fuzzy normal $M$-subgroup of a $M$-group $G$.

**Proof:** Let $x$ and $y$ in $G$ and $A$ be an intuitionistic anti $L$-fuzzy $M$-subgroup of a $M$-group $H$.

We know that, $A^f$ is an intuitionistic anti $L$-fuzzy $M$-subgroup of a $M$-group $G$.

Then we have, 

$$\mu_{A^f}(xy) = \mu_A(f(xy))$$

which implies that

$$(\mu_{A^f})(xy) = (\mu_A)(yx), \text{ for all } x \text{ and } y \text{ in } G.$$ 

Now, 

$$\nu_{A^f}(xy) = \nu_A(f(xy))$$

which implies that

$$(\nu_{A^f})(xy) = (\nu_A)(yx), \text{ for all } x \text{ and } y \text{ in } G.$$ 

Hence $(A^f)$ is an intuitionistic anti $L$-fuzzy normal $M$-subgroup of a $M$-group $G$.

**V CONCLUSION**

Further work is in progress in order to develop the intuitionistic anti $L$-fuzzy normal $M$-subgroups, intuitionistic anti $L$-fuzzy $N$-subgroups and intuitionistic anti $L$-fuzzy normal $M$-subgroups.

**REFERENCES**


