Intuitionistic Fuzzy Basis in Topological Spaces

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Abstract
In this paper, the notion of intuitionistic fuzzy basis and strong intuitionistic fuzzy basis is introduced. Theorems related to crisp basis, intuitionistic fuzzy basis and strong intuitionistic fuzzy basis are stated and proved.

I. INTRODUCTION
In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets. The notion of defining intuitionistic fuzzy sets (IFSs) for fuzzy set generalizations, introduced by Atanassov[1], has proven interesting and useful in various application areas. Since this fuzzy set generalization can present the degrees of membership and non-membership with a degree of hesitancy, the knowledge and semantic representation becomes more meaningful and applicable. Chang (1968) [3] was the first to introduce the concept of a fuzzy topology. Muthukumari et al., initiated the concept of fuzzy basis. A.M.Ali et al., introduced the concept of intuitionistic fuzzy set in metric spaces. In this paper, the concept of intuitionistic fuzzy basis and strong intuitionistic fuzzy basis is introduced. Some results regarding to crisp basis, intuitionistic fuzzy basis and strong intuitionistic fuzzy basis are investigated.

II. PRELIMINARIES

Definition 2.1
Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a non-empty finite set A fuzzy set \( F \) of \( X \) can be defined as \( F= \{(x,F(x))|\ \forall x \in X\} \); where \( F(x):X \to [0, 1] \) is the degree of membership of \( x \) in \( X \).

Definition 2.2
A fuzzy topology is a family \( T \) of fuzzy sets in \( X \) which satisfies the following conditions:
1. \( \emptyset, X \in T \),
2. If \( A, B \in T \) then \( A \cap B \in T \),
3. If \( A_i \in T \) for each \( i \in I \), then \( \bigcup_{i \in I} A_i \in T \). The pair \( (X,T) \) is called a fuzzy topological space.

Definition 2.3
Let \( X \) be a non empty set. \( B \subset P(X) \) is called a base if

1. \( \cup \{B/B \in B\} = X \),
2. \( U \cup V \in B \) and \( x \in U \cap V \) implies there exists \( W \in B \) such that \( x \in W \subset U \cap V \).
Let \( T \) be the collection of all union of finite number of elements of \( B \). Then \( T \) is a topology and \( B \) is a base for the topology.

Definition 2.4
Let \( X \) be a nonempty set. A function \( f: P(X) \to [0,1] \) is called a fuzzy basis if
1. \( \cup \{B/B \in B\} = X \).
2. For each \( \alpha \in (0,1] \) \( f(U) \geq \alpha, f(V) \geq \alpha \) and \( x \in U \cap V \) implies there exist \( W \) with \( f(W) \geq \alpha \) and \( x \in W \subset U \cap V \).

Definition 2.5
Let \( X \) be a nonempty set. \( B \subset P(X) \) is called a strong crisp basis if
1. \( \cup \{B/B \in B\} = X \).
2. \( U, V \in B, U \cap V \neq \emptyset \Rightarrow U \cup V \in B \).

Example 2.1
1. \( X=\{a,b,c\} \), \( B=\{\{a,b\},\{b,c\},\{b\}\} \)
2. \( X=\{a,b,c,d\} \), \( B=\{\{a,b,c\},\{b,c,d\},\{b,c\}\} \).

Definition 2.6
An intuitionistic fuzzy set \( F \) in \( X \) can be formulated as \( F = \{(x, \mu_F(x), \nu_F(x)) | \forall x \in X \} \) where \( \mu_F(x), \nu_F(x) : X \to [0,1] \) represent the degree of membership and non-membership of \( x \) in \( X \), respectively, with the essential condition \( 0 \leq \mu_F(x) + \nu_F(x) \leq 1 \).

III. INTUITIONISTIC FUZZY BASIS

Definition 3.1
Let \( X \) be a nonempty set. A function \( (f_\mu, f_\nu) : P(X) \to [0,1] \) is called an intuitionistic fuzzy basis if
1. \( \cup \{B/B \in B\} = X \).
2. For each \( (\alpha, \beta) \in (0,1] \) with \( \alpha + \beta \leq 1 \), \( f_\mu(U) \geq \alpha, f_\nu(U) \leq \beta \) and \( f_\mu(V) \geq \alpha, f_\nu(V) \leq \beta \) and \( x \in U \cap V \) implies there exist \( W \) with \( f_\mu(W) \geq \alpha, f_\nu(W) \leq \beta \) and \( x \in W \subset U \cap V \).

Example 3.1
Let \( X=\{a,b,c,d\} \), \( (f_\mu, f_\nu) : P(X) \to [0,1] \) as
\[ f_\alpha (X) = 1, \ f_\beta (X) = 0, \]
\[ f_\alpha (\{a, b, c\}) = 1, \ f_\alpha (\{a, b, c\}) = 0, \]
\[ f_\alpha (\{b, c, d\}) = 1, \ f_\alpha (\{b, c, d\}) = 0, \]
\[ f_\alpha (\{b\}) = 1, \ f_\alpha (\{b\}) = 0, \]
\[ f_\alpha (\{c\}) = 1, \ f_\alpha (\{c\}) = 0, \]
\[ f_\alpha (\{d\}) = 0.6, \ f_\alpha (\{a, b, c\}) = 0.2, \]
\[ f_\alpha (\{a, b, c\}) = 0.6, \ f_\alpha (\{a, b, c\}) = 0.3, \]
\[ f_\alpha (\{b\}) = 0.4, \ f_\alpha (\{a\}) = 0.1, \]
\[ f_\alpha (\{d\}) = 0.5, \ f_\alpha (\{d\}) = 0.2, \]
\[ f_\alpha (\{A\}) = 0, \ f_\alpha (\{A\}) = 1. \]

Claim: \( f_\alpha (f_\beta (X)) \geq \alpha \) and \( f_\beta (f_\alpha (X)) \geq \beta \).

\[ \text{The fuzzy basis induced by a strong crisp basis is a strong intuitionistic fuzzy basis.} \]

Example 3.3

Let \( X = \{a, b, c, d\} \).

Define \( f_\alpha (X) \rightarrow [0,1] \) as

\[ f_\alpha (X) = 1, \ f_\beta (X) = 0. \]

Claim: \( f_\alpha (f_\beta (X)) \geq \alpha \) and \( f_\beta (f_\alpha (X)) \geq \beta \).

\[ \text{The fuzzy basis induced by a strong crisp basis is a strong intuitionistic fuzzy basis.} \]
0, \( f_\mu(U) = 1 \ f_\mu(V) = 1, f_\mu(V) = 0 \) then whatever be the value of \((U \cap V)\), we have \( f_\mu(U \cap V) \geq \min\{ f_\mu(U), f_\mu(V)\} \leq \max\{ f_\mu(U), f_\mu(V)\} \). If \( f_\mu(U) = 1, f_\mu(U) = 0 \) or \( f_\mu(V) = 1, f_\mu(V) = 0 \) then \( U, V \in B \). Since \( B \) is a strong basis, \( U \cap V \in B \). Hence \( f_\mu(U \cap V) \geq 1, f_\mu(U \cap V) \leq 0 \).

Hence \( f_\mu(U \cap V) \geq \min\{ f_\mu(U), f_\mu(V)\} \leq \max\{ f_\mu(U), f_\mu(V)\} \).

Hence \( f \) is a strong intuitionistic fuzzy basis.

REFERENCES