Some New kinds of Connected Domination in Intuitionistic Fuzzy Graphs

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Abstract: In this paper, we introduce the concept of some new kinds of connected domination number of an intuitionistic fuzzy graphs. We determine the domination numbers γνΦ, γνΦa, γνβ, γνα and the total domination number of γνΦ for several classes of intuitionistic fuzzy graphs and obtain bounds for the same. We also obtain the Nordhaus – Gaddum type result for these parameters.

Keywords: Connected strong domination number, disconnected strong domination number, total disconnected strong domination number, left semi connected domination number, right semi connected domination number.

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INTRODUCTION

The study of domination set in graphs was begun by Ore and Berge. The connected domination number was first introduced by E.Sampathkumar and H.B.Walikar[8] Rosenfield[6] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness. A Somasundram and S.Somasundaram[10] discussed domination in fuzzy graphs. K.T. Atanassov[1] initiated the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. R.Parvathi and M.G. Karunambigai[6] gave a definition of IFG as a special case of IFGS defined by K.T Atanassov and A.Shannon. R.Parvathi and G.Thamizhendhi[7] was introduced dominating set and domination number in IFGS. In this paper, we discuss some new kinds of connected domination number of an intuitionistic fuzzy graphs and obtain the relationship with other known parameters of an IFG G.

II. PRELIMINARIES

Definition 2.1
An Intuitionistic Fuzzy Graph (IFG) is of the form G = (V, E) where
(i) V = {V1, V2, …, Vn} such that σ1 : V → [0, 1] and σ2 : V → [0, 1] denote the degree of membership and non-membership of the element vi ∈ V respectively and 0 ≤ σ1(vi) + σ2(vi) ≤ 1, for every vi ∈ V
(ii) E ⊆ V x V where μ1 : V x V → [0, 1] and μ2 : V x V → [0, 1] are such that μ1(vi, vj) = min {σ1(vi), σ1(vj)} μ2(vi, vj) = max {σ2(vi), σ2(vj)} and 0 ≤ μ1(vi, vj) + μ2(vi, vj) ≤ 1 for every (vi, vj) ∈ E.

Note: when μ1ij = μ2ij=0 for some i and j then there is no edge between vi and vj otherwise there exits an edge between vi and vj.

Definition 2.2
An IFG H = (V', E') is said to be an IF subgraph (IFSG) of G = (V, E) if V' ⊆ E and E' ⊆ E. That is σ1i' ≤ σ1i ; σ2i' ≥ σ2i and μ1i' ≤ μ1i ; μ2i' ≥ μ2i, for every i = 1, 2, …, n

Definition 2.3
The intuitionistic fuzzy subgraph H = (V', E') is said to be a spanning fuzzy subgraph of an IFG G = (V, E) if σ1' (u) = σ1 (u) and σ2' (u) = σ2 (u) for all u ∈ V' and μ1i'(u, v) ≤ μ1i(u, v) and μ2i'(u, v) ≥ μ2i(u, v) for all u,v ∈ V.

Definition 2.4
Let G = (V, E) be an IFG. Then the vertex cardinality of G is defined by

Definition 2.5
Let G = (V, E) be an IFG. Then the edge cardinality of E is defined by
Definition 2.6
Let \( G = (V, E) \) be an IFG. Then the cardinality of \( G \) is defined to be \( |G| = |V| + |E| = p + q \)

Definition 2.7
The number of vertices is called the order of an IFG and is denoted by \( O(G) \). The number of edge is called size of IFG and is denoted by \( S(G) \).

Definition 2.8
The vertices \( v_i \) and \( v_j \) are said to be connected in IFG either one of the following conditions hold

(i) \( \mu_1 (v_i, v_j) > 0, \mu_2 (v_i, v_j) > 0 \)
(ii) \( \mu_1 (v_i, v_j) = 0, \mu_2 (v_i, v_j) > 0 \)
(iii) \( \mu_1 (v_i, v_j) > 0, \mu_2 (v_i, v_j) = 0, v_i, v_j \in V \)

Definition 2.9
A path in an IFG is a sequence of distinct vertices \( v_1, v_2, \ldots, v_n \) such that either one of the following conditions is satisfied.

(i) \( \mu_1 (v_i, v_j) > 0, \mu_2 (v_i, v_j) > 0 \) for some \( i, j \)
(ii) \( \mu_1 (v_i, v_j) = 0, \mu_2 (v_i, v_j) > 0 \) for some \( i, j \)
(iii) \( \mu_1 (v_i, v_j) > 0, \mu_2 (v_i, v_j) = 0 \) for some \( i, j \)

Note: The length of the path \( P = v_1, v_2, \ldots, v_{n+1} \) is \( n > 0 \)

Definition 2.10
Two vertices that are joined by a path is called connected.

Definition 2.11
An edge \( e = (x, y) \) of an IFG \( G = (V, E) \) is called an effective edge if \( \mu_1 (x, y) = \min \{ \sigma_1(x), \sigma_1(y) \} \) and \( \mu_2 (x, y) = \max \{ \sigma_2(x), \sigma_2(y) \} \).

Definition 2.12
An IFG \( G = (V, E) \) is said to be complete IFG if \( \mu_{ij} = \min \{ \sigma_{1i}, \sigma_{1j} \} \) and \( \mu_{2j} = \max \{ \sigma_{2i}, \sigma_{2j} \} \) for every \( v_i, v_j \in V \).

Definition 2.13
The complement of an IFG, \( G = (V, E) \) is an IFG, \( \overline{G} = (V, \overline{E}) \), where

\[
\begin{align*}
(i) & \quad \overline{V} = V \\
(ii) & \quad \overline{\sigma_i} = \sigma_i \text{ and } \overline{\sigma_2} = \sigma_2, \text{ for all } i = 1, 2, \ldots, n \\
(iii) & \quad \overline{\mu_{ij}} = \min \{ \sigma_{1i} - \sigma_{1j}, \sigma_{2i} - \sigma_{2j} \} \\
(iv) & \quad \overline{\mu_{2i}} = \max \{ \sigma_{2i}, \sigma_{2j} \} - \mu_{2j} \text{ for all } i = 1, 2, \ldots, n
\end{align*}
\]

Definition 2.14
An IFG, \( G = (V, E) \) is said to be bipartite if the vertex set \( V \) can be partitioned into two non empty sets \( V_1 \) and \( V_2 \) such that

(i) \( \mu_1 (v_i, v_j) = 0 \) and \( \mu_2 (v_i, v_j) = 0 \) if \( v_i, v_j \in V_1 \) (or) \( v_i, v_j \in V_2 \)
(ii) \( \mu_1 (v_i, v_j) > 0 \) and \( \mu_2 (v_i, v_j) > 0 \) if \( v_i \in V_1 \) and \( v_j \in V_2 \) for some \( i, j \)
(iii) \( \mu_1 (v_i, v_j) = 0 \) and \( \mu_2 (v_i, v_j) > 0 \) if \( v_i \in V_1 \) and \( v_j \in V_2 \) for some \( i, j \)

Definition 2.15
A bipartite IFG, \( G = (V, E) \) is said to be complete if

\[
\begin{align*}
\mu_1 (v_i, v_j) = \min \{ \sigma_1(v_i), \sigma_1(v_j) \} \\
\mu_2 (v_i, v_j) = \max \{ \sigma_2(v_i), \sigma_2(v_j) \} \\
\text{for all } v_i \in V_1 \text{ and } v_j \in V_2.
\end{align*}
\]

It is denoted by \( K(\sigma_1, \mu_1, \sigma_2, \mu_2) \).

Definition 2.16
A vertex \( u \in V \) of an IFG \( G = (V, E) \) is said to be an isolated vertex if \( \mu_1 (u, v) = 0 \) and \( \mu_2 (u, v) = 0 \) for all \( v \in V \). That is \( N(u) = \phi \). Thus an isolated vertex does not dominate any other vertex in \( G \).

Definition 2.17
Let \( G = (V, E) \) be an IFG on \( V \). Let \( u, v \in V \), we say that \( u \) dominates \( v \) in \( G \) if there exists a edge between them.

Definition 2.18
Let \( G = (V, E) \) be an intuitionistic fuzzy graph \( G \) on the vertex set \( V \). Let \( x, y \in V \), we say that \( x \) dominates \( y \) in \( G \) if \( \mu_1 (x, y) = \min \{ \sigma_1(x), \sigma_1(y) \} \) and \( \mu_2 (x, y) = \max \{ \sigma_2(x), \sigma_2(y) \} \). A subset \( D \) of \( V \) is called a dominating set in IFG if for every \( v \in V - D \), there exists \( u \in D \) such that \( u \) dominates \( v \) in a dominating set \( D \) of an IFG is said to be minimal dominating set if no proper subset of \( D \) is a dominating set.

III. INTUITIONISTIC FUZZY CONNECTED STRONG DOMINATION NUMBER

Definition 3.1
Let \( G = (V, E) \) be a IFG without isolated vertices. A subset \( D_{i+1} \) of \( V \) is said to be an intuitionistic fuzzy connected strong domination set if both induced subgraphs \( \langle D_{i+1} \rangle \) and \( \langle V - D_{i+1} \rangle \) are connected. The intuitionistic fuzzy connected strong domination number \( \gamma_{c}(G) \) is the minimum intuitionistic fuzzy cardinality taken over all connected strong dominating sets of \( G \).
Example 3.1.1

Let G = (V,E) be an IFG. A subset Dcs of V is said to be an intuitionistic fuzzy total connected strong domination set if

\[ Dcs \in Dcs(G) \Rightarrow \forall v \in V, \exists v \neq u \in Dcs : v \neq u \] 

The intuitionistic fuzzy total connected strong domination number \( \gamma_{tcs}(G) \) is the minimum intuitionistic fuzzy cardinality taken over all total connected strong dominating sets in G.

Proposition 3.1

\[ \gamma_{tcs}(P_5) = \min \{ 1 + [ p_1- \sigma_1(v_1) - p_2- \sigma_2(v_1) ] , \frac{1}{2} [ 1 + [ p_1- \sigma_1(v_2) - p_2- \sigma_2(v_2) ] \} \]

Proposition 3.2

\[ \gamma_{tcs}(C_n) = \begin{cases} \frac{1}{2} \left( 1 + \min \sum_{i=1}^{n} \sigma_i(v_i) , \sum_{i=1}^{n-1} \sigma_i(v_i) , \ldots , \sum_{i=n-2}^{1} \sigma_i(v_i) \right) & \text{max} \sum_{i=1}^{n} \sigma_i(v_i) \end{cases} \]

Proposition 3.3

\[ \gamma_{tcs}(w_0) = \frac{1}{2} [ 1 + \sigma_1(v) - \sigma_2(v) ] , v \text{ is the centre vertex } |v| \]

Proposition 3.4

\[ \gamma_{tcs}(k_0) = \frac{1}{2} [ 1 + \sigma_1(v) - \sigma_2(v) ] , v \text{ is the vertex of minimum cardinality.} \]

Proposition 3.5

\[ \gamma_{cs}(v) = \begin{cases} \frac{1}{2} ( 1 + \sigma_1(v) - \sigma_2(v) ) & \frac{1}{2} ( 1 + \sigma_1(v) - \sigma_2(v) ) + \frac{1}{2} ( 1 + \sigma_1(v) - \sigma_2(v) ) \end{cases} \]

\[ = |v| + |v| , v \text{ is a vertex adjacent with all other vertices and } v_i \text{ is the all pendent vertices of minimum cardinality, except one pendent vertex has maximum cardinality.} \]

Proposition 3.6

\[ \gamma_{cs}(K_{n+1, n}) = \min \{ |v_1| + |v_2| \} \text{ where } v_1 \in V_1 \text{ and } v_2 \in V_2. \]

\[ |v| = \frac{1}{2} ( 1 + \sigma_1(v) - \sigma_2(v) ) \]

\[ \gamma_{tcs}(G) = \min \left\{ \sum_{i=1}^{n} |v_i| , \sum_{i=1}^{n} |v_i| \right\} \]

\[ \gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G) \]

\[ \gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G) \]

Example 3.2.1

Let G be the Peterson graph, (i) If all intuitionistic fuzzy vertices having equal membership and non-membership value then

\[ \gamma_{cs}(G) = 5|v| = 5/2 ( 1 + \sigma_1(v_i) - \sigma_2(v_i) ) , i = 1 \to 10 \]

(ii) If an unequal intuitionistic fuzzy vertex cardinality then,

\[ \gamma_{tcs}(G) = \min \left\{ \sum_{i=1}^{n} |v_i| , \sum_{i=1}^{n} |v_i| \right\} \]

\[ \gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G) \]

\[ \gamma_{cs}(G) \leq \gamma_{tcs}(G) \leq 2\gamma_{cs}(G) \]
\[ \sigma_1(v_i) = 0.1, \sigma_2(v_i) = 0.1, \text{for all } v_i \in V \\
\mu_1(v_i,v_j) = 0.1, \mu_2(v_i,v_j) = 0.1, \text{for all } (v_i,v_j) \in E \\
\]

\[
D_{cs} = \{v_2,v_3\}, \gamma_{cs} = 0.5 \\
D_{cs} = \{v_2,v_3\}, \gamma_{cs} = 0.5 \\
\therefore \gamma_{cs}(G) \leq \gamma_{cs}(G) \leq 2\gamma_{cs}(G)
\]

**Theorem 3.3**

If \( G = (V,E) \) is a connected intuitionistic fuzzy graph, then,

\[
\gamma_{cd}(G) \leq |V| - \left\lceil \frac{1}{2} \left( 1 + \sigma_1(v_i) - \sigma_2(v_i) \right) \right\rceil - \left\lfloor \frac{1}{2} \right\rfloor
\]

\[= |v| - \frac{1}{2} \]

\[\{2+(\sigma_1(v_i) + \sigma_2(v_i)) - (\sigma_1(v_i) + \sigma_2(v_i))\} \]

Where \( v_i,v_j \) are the vertex having first two maximum intuitionistic fuzzy cardinality among the all vertices.

**Theorem 3.4**

If \( G = (V,E) \) is a fuzzy graph then

\[ |v| \leq \gamma_{cd}(G) \leq |V| - |v| \]

\[\text{ii) } 2|v| \leq \gamma_{cs}(G) - |v| \leq |V|, v_i \text{ is the vertices of intuitionistic fuzzy minimum cardinality.} \]

**Proof**:

Let \( G = (V,E) \) be an IFG.

By definition of intuitionistic fuzzy dominating set, \( \gamma(G) \leq |V| \).

Clearly \( \gamma(G) \leq \gamma_{cd}(G) \).

Suppose all fuzzy vertices are isolated then \( \gamma(G) = |V| \).

Clearly \( \gamma_{cd}(G) < |V| \).

Therefore, \( |v| \leq \gamma_{cd}(G) \) when \( G \) is complete or not

\[\therefore |v| \leq \gamma_{cd}(G) \leq |V| - |v| \]

**Theorem 3.5**

If \( G = (V,E) \) is an intuitionistic fuzzy path, all connected strong dominating set are minimal dominating sets.

**Proof**:

By theorem 2.5, \( G \) has exactly two different connected strong dominating sets.

\[D_1 = \{v_1,v_2,...,v_n\} \]

\[D_2 = \{v_2,v_3,v_4,...,v_n\} \]

\[\text{Obviously } D_{cs}(v_i) \text{ is not a connected strong dominating set, for all } v_i \in D_1. \]

\[\text{Hence } D_1 \text{ is a minimal connected strong dominating set. Similarly for } D_2. \]

\[\therefore \text{Both } D_1 \text{ and } D_2 \text{ are minimal connected strong dominating sets of an IFG.} \]

**Theorem 3.6**

If \( G = (V,E) \) is an intuitionistic fuzzy path then \( G \) has exactly two connected strong dominating set.

**Proof**:

Let \( V = \{v_1,v_2,...,v_n\} \) be an intuitionistic fuzzy vertex set of \( G \), by definition 3.1.

Clearly \( D_1 = \{v_1,v_2,...,v_{n-1}\} \) and \( D_2 = \{v_2,v_3,...,v_n\} \) are the two intuitionistic fuzzy connected strong dominating sets.

**IV. INTUITIONISTIC FUZZY DISCONNECTED STRONG DOMINATING NUMBER**

**Definition 4.1**

Let \( G = (V,E) \) be an IFG without isolated vertices. A subset \( D_{cs} \) of \( V \) is said to be an intuitionistic fuzzy disconnected strong dominating set if both induced subgraphs \( <D_{cs}> \) and \( <V-D_{cs}> \) are disconnected. The intuitionistic fuzzy disconnected strong domination number \( \gamma_{ds}(G) \) is the minimum intuitionistic fuzzy cardinality taken over all disconnected strong dominating sets of \( G \).

**Example 4.1**

\[
(0.1, 0.1)_{v_7} \]

If \( \sigma_1(v_i) = 0.1, \sigma_2(v_i) = 0.1, \text{for all } v_i \in V \\
\mu_1(v_i,v_j) = 0.1, \mu_2(v_i,v_j) = 0.1, \text{for all } (v_i,v_j) \in E \\
D_{cs} = \{v_7\}, V - D_{cs} = \{v_1,v_2,v_3,v_4,v_5,v_6\} \\
<\text{Disconnected}> \text{ and } <\text{V-D}_{cs}> \text{ are disconnected.} \\
\gamma_{ds}(G) = 1
\]

**Proposition : 4.1**

\[
\gamma_{ds}(P_n) = \left[ \frac{n}{2} \right] \text{vi} \text{ having equal intuitionistic fuzzy cardinality, } n \text{ is the order of an IFG.}
\]

**Proposition : 4.2**

\[
\gamma_{ds}(C_n) = \left[ \frac{n}{3} \right] \text{vi} \text{ having equal intuitionistic fuzzy cardinality, } n \geq 4.
\]

**Proposition : 4.3**

\[
\gamma_{ds}(K_{n+2,2,m \mu_{1,2}}) = \min \left\{ \sum_{i=1}^{n} \left| v_i \right|, \sum_{j=1}^{m} \left| v_j \right| \right\}
\]

Where \( v_i \in V_1 \text{and } v_j \in V_2. \)

**Theorem 4.1**
For any IFG, \( G = (V, E) \)
\[ |V| - |E| \leq \gamma_{lsd}(G) \leq |V| - \Delta \]

**Proof**: Let \( V \) be a vertex of an intuitionistic fuzzy graph, such that \( dN(v) = \Delta \), then \( V/N(v) \) is a dominating set of an intuitionistic fuzzy graph \( G \).

So that \( \gamma_{lsd}(G) \leq |V/N(v)| = |V| - \Delta \)

**Definition 4.2**: Let \( G(V,E) \) be an IFG G. A subset \( D_{ds} \) of \( V \) is said to be an intuitionistic fuzzy total disconnected strong dominating set if

i) \( D_{ds} \) is disconnected strong dominating set

ii) \( N[D_{ds}] = V \)

The intuitionistic fuzzy total disconneced strong dominating number \( \gamma_{ds} \) is the minimum cardinality taken overall all total disconnected strong dominating set in \( G \).

**V. INTUITIONISTIC FUZZY LEFT SEMI CONNECTED DOMINATION NUMBER**

**Definition 5.1**: Let \( G = (V,E) \) be an intuitionistic fuzzy graph without isolated vertices. A subset \( D_{lsd} \) of \( V \) is said to be a fuzzy left semi connected strong dominating set if the induced intuitionistic fuzzy subgraph \( <D_{lsd}> \) is connected and induced intuitionistic fuzzy subgraph \( <V-D_{lsd}> \) is disconnected.

The intuitionistic fuzzy left semi connected domination number \( \gamma_{lsd}(G) \) is the minimum intuitionistic fuzzy cardinality taken overall left semi connected dominating sets of \( G \).

**Example 5.1**

\[ D_{lsd} = \{v_1,v_4\}, \]
\[ <D_{lsd}> \text{ is connected} \]
\[ <V-D_{lsd}> \text{ is disconnected} \]
\[ \gamma_{lsd}(G) = 1.0 \]

**Proposition 5.1**
\[ \gamma_{lsd}(P_3) = |V| - (|v_1| + |v_2|) \]

**Proposition 5.2**
\[ \gamma_{lsd}(W_4) = 3|v_1|, \text{ all } v_i's \text{ having equal intuitionistic fuzzy cardinality.} \]

**Proposition 5.3**
\[ \gamma_{lsd}(\mathcal{E}_5) = |v_1|, \text{ where } v_i \text{ is the intuitionistic fuzzy vertex having maximum effective degree.} \]

**Proposition 5.4**
\[ \gamma_{lsd}(K_{11}) = \min \left\{ \sum_{i=1}^{n} |v_i|, j = 1 to n \right\} \]

**Theorem 5.1**
For any intuitionistic fuzzy graph \( G, \gamma(G) \leq \gamma_{lsd}(G) \)

**Theorem 5.2**
Let \( G = (V,E) \) be an intuitionistic fuzzy connected graph, and \( H = (V',E') \) be an intuitionistic spanning fuzzy subgraph of \( G \), if \( H \) has a left semi connected dominating set then \( \gamma_{lsd}(G) \leq \gamma_{lsd}(H) \).

**Theorem 5.3**
For any intuitionistic fuzzy graph \( G = (V,E), \gamma_{lsd}(G) + \gamma_{lsd}(\overline{G}) \leq 2|V|, \) where \( \gamma_{lsd}(\overline{G}) \) is the left semi connected domination number of \( \overline{G} \) and equality holds if \( \forall u,v \) \( \mu_{1}(u,v) < \tau_{1}(u) \land \tau_{2}(v) \) and \( 0 \leq \mu_{2}(u,v) \lor \tau_{2}(v) \) for all \( u,v \in V \).

**VI. INTUITIONISTIC FUZZY RIGHT SEMI CONNECTED DOMINATION NUMBER**

**Definition 6.1**: Let \( G = (V,E) \) be an intuitionistic fuzzy graph without isolated vertices. A subset \( D_{rsd} \) of \( V \) is said to be an intuitionistic fuzzy right semi connected dominating set if the induced intuitionistic fuzzy subgraph \( <D_{rsd}> \) is connected and induced intuitionistic fuzzy subgraph \( <V-D_{rsd}> \) is disconnected.

The intuitionistic fuzzy right semi connected domination number \( \gamma_{rsd}(G) \) is the minimum intuitionistic fuzzy cardinality taken overall right semi connected dominating sets of \( G \).

**Example 6.1**

\[ D_{rsd} = \{v_1,v_4\}, \]
\[ V-D_{rsd} = \{v_2,v_3,v_5,v_6,v_7\} \]
\[ <D_{rsd}> \text{ is disconnected and} \]
\[ <V-D_{rsd}> \text{ is connected} \]
\[ \gamma_{rsd}(G) = 0.95 \]
Proposition 6.1
\[ \gamma_{\text{rsc}}(P_n) = \min \left\{ \sum_{i=1}^{n-3} |v_i| + \left| \sum_{i=1}^{n} |v_i| + |v_1| \right| \right\} \]

Proposition 6.2
\[ \gamma_{\text{rsc}}(P_n) = \left\lfloor \frac{n}{3} \right\rfloor |v_1| ; \forall v_i \text{'s having equal intuitionistic fuzzy cardinality} \]

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