Homomorphism and Anti-Homomorphism of an Intuitionistic Anti L-Fuzzy Translation

Dr. P Pandiammal, Vinotha

Department of Mathematics, PSNACET, Dindigul, Tamil Nadu, India

Department of Mathematics, PSNACET, Dindigul, Tamil Nadu, India

Abstract

This paper contains some definitions and results in intuitionistic anti L-fuzzy translation of intuitionistic anti L-fuzzy M-subgroup of a M-group, which are required in the sequel. Some properties of homomorphism and anti-homomorphism of intuitionistic anti L-fuzzy translation are also established.

Keywords:

Introduction:
The notion of fuzzy sets was introduced by L.A. Zadeh [12]. Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, Rosenfield [1] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. In [2], Biswas introduced the concept of anti-fuzzy subgroups of groups. Palaniappan, N and Muthuraj, [8] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. Pandiammal, P, Natarajan, R, and Palaniappan, N, [10] defined the homomorphism, anti-homomorphism of an anti L-fuzzy M-subgroup. Kandasamy[4] introduced the concept of fuzzy translation and fuzzy multiplication. The idea of fuzzy magnified translation has been introduced by Majumder and Sardar [5]. Pandiammal, P [11] defined the concept of Intuitionistic Anti L-fuzzy M-subgroups. In this paper we define a new algebraic structure of intuitionistic anti L-fuzzy translation of intuitionistic anti L-fuzzy M-subgroup of an M-group and study some their related properties.

1. Preliminaries:

Intuitionistic Anti L-Fuzzy M-Subgroups

1.1 Definition: Let (G, ·) be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an intuitionistic L-fuzzy M-subgroup (ILFMSG) of G if the following conditions are satisfied:

(i) \( \mu_A( mxy ) \geq \mu_A(x) \land \mu_A(y) \),
(ii) \( \mu_A( x^{-1} ) \geq \mu_A( x ) \),
(iii) \( \nu_A( mxy ) \leq \nu_A(x) \lor \nu_A(y) \),
(iv) \( \nu_A( x^{-1} ) \leq \nu_A( x ) \),

for all x and y in G.

1.2 Definition: An intuitionistic fuzzy subset \( \mu \) in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following axioms are satisfied.

(i) \( \mu_A( xy ) \leq \mu_A(x) \lor \mu_A(y) \),
(ii) \( \mu_A( x^{-1} ) \leq \mu_A( x ) \),
(iii) \( \nu_A( xy ) \geq \nu_A(x) \land \nu_A(y) \),
(iv) \( \nu_A( x^{-1} ) \geq \nu_A( x ) \),

for all \( x, y \) in G.

1.3 Proposition: Let G be a group. An intuitionistic fuzzy subset \( \mu \) in a group G is said to be an intuitionistic anti fuzzy subgroup of G if the following conditions are satisfied,

(i) \( \mu_A( xy ) \leq \mu_A(x) \lor \mu_A(y) \),
(ii) \( \nu_A( xy ) \geq \nu_A(x) \land \nu_A(y) \),

for all \( x, y \) in G.

1.4 Definition: Let G be an M-group and \( \mu \) be an intuitionistic anti fuzzy group of G. If \( \mu_A( mx ) \leq \mu_A( x ) \) and \( \nu_A( mx ) \geq \nu_A( x ) \) for all x in G and m in M then \( \mu \) is said to be an intuitionistic anti fuzzy subgroup with operator of G. We use the phrase \( \mu \) is an intuitionistic anti L-fuzzy M-subgroup of G.
1.5 Example: Let H be M-subgroup of an M-group G and let A = (μA, νA) be an intuitionistic fuzzy set in G defined by
\[ (x; x \in H) \]
\[ \mu_A(x) = \begin{cases} 
0.5; & \text{otherwise} \\
0.6; & x \in H 
\end{cases} \]
\[ \nu_A(x) = \begin{cases} 
0.3; & \text{otherwise} 
\end{cases} \]
For all x in G. Then it is easy to verify that A = (μA, νA) is an anti-fuzzy M-subgroup of G.

1.6 Definition: Let A be an intuitionistic L-fuzzy subset of X and α and β in [0, 1].
\[ 1 - Sup\{ \mu_A(x) + \nu_A(x) : x \in X, 0 < \mu_A(x) + \nu_A(x) < 1 \} \]. Then T = \( T_{(\alpha, \beta)}^A \) is called an intuitionistic L-fuzzy translation of A if \( \mu_T(x) = \mu_A(x) + \alpha \), \( \nu_T(x) = \nu_A(x) + \beta \), \( \alpha + \beta \leq 1 \), \( \mu_A(x) + \nu_A(x) < 1 \), for all x in X.

2 - PROPERTIES OF INTUITIONISTIC ANTI L-FUZZY TRANSLATION:

2.1 Theorem: If T is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup of a M-group G, then \( \mu_T(x^{-1}) = \mu_T(x) \) and \( \nu_T(x^{-1}) = \nu_T(x) \), \( \mu_T(x) \leq \mu_T(e) \) and \( \nu_T(x) \leq \nu_T(e) \), for all x and e in G.

Proof: Let x and e be elements of G. Now,
\[ \mu_T(x) = \mu_A(x) + \alpha \]
\[ = \mu_A(x^{-1}) + \alpha \]
\[ \leq \mu_A(x) + \alpha \]
\[ = \mu_T(x^{-1}) \]
\[ = \mu_A(x^{-1}) + \alpha \]
\[ \leq \mu_A(x) + \alpha = \mu_T(x). \]
Therefore, \( \mu_T(x) = \mu_T(x^{-1}) \), for x in G.

And, \( \nu_T(x) = \nu_A(x) + \beta \)
\[ = \nu_A(x^{-1}) + \beta \]
\[ \geq \nu_A(x^{-1}) + \beta \]
\[ = \nu_T(x) \]
\[ = \nu_A(x^{-1}) + \beta \]
\[ \geq \nu_A(x) + \beta = \nu_T(x). \]
Therefore, \( \nu_T(x) = \nu_T(x^{-1}) \), for x in G.

Now, \( \mu_T(e) = \mu_A(e) + \alpha \)
\[ = \mu_A(xx^{-1}) + \alpha \]
\[ \leq \{ \mu_A(x) \lor \mu_A(x^{-1}) \} + \alpha \]
\[ = \mu_A(x) + \alpha \]
Therefore, \( \mu_T(e) = \mu_T(x) \) for x in G.
And \( \nu_T(e) = \nu_A(e) + \beta \)
\[ = \nu_A(xx^{-1}) + \beta \]
\[ \geq \{ \nu_A(x) \land \nu_A(x^{-1}) \} + \beta \]
\[ = \nu_A(x) + \beta \]
Therefore, \( \nu_T(e) = \nu_T(x) \) for x in G.

2.2 Theorem: If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy M-subgroup A of a M-group G, then (i) \( \mu_T(xy^{-1}) = \mu_T(x) \) implies \( \mu_T(y) = \mu_T(y) \), (ii) \( \nu_T(xy^{-1}) = \nu_T(e) \) implies \( \nu_T(x) = \nu_T(y) \), for all x, y and e in G.

Proof: Let x, y and e be elements of G. Now,
\[ \mu_T(x) = \mu_A(x) + \alpha \]
\[ = \mu_A(xy^{-1}) + \alpha \]
\[ \leq \{ \mu_A(xy^{-1}) + \alpha \} \lor \{ \mu_A(y^{-1}) + \alpha \} \]
\[ = \mu_T(x) \lor \mu_T(y) \]
Therefore, \( \mu_T(x) = \mu_T(y) \), for all x and y in G.
And \( \nu_T(x) = \nu_A(x) + \beta \)
\[ = \nu_A(xy^{-1}) + \beta \]
\[ \geq \{ \nu_A(xy^{-1}) + \beta \} \land \{ \nu_A(y^{-1}) + \beta \} \]
\[ = \nu_T(x) \land \nu_T(y) \]
Therefore, \( \nu_T(x) = \nu_T(y) \), for all x and y in G.

2.3 Theorem: If T is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup A of a M-group G, then T is an intuitionistic anti L-fuzzy M-subgroup of a M-group G, for all x and y in G.

Proof: Assume that T is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup A of a M-group G. Let x and y in G. We have
\[ \mu_T(mxy^{-1}) = \mu_A(mxy) + \alpha \]
\[ \leq (\mu_A(x) \land \mu_A(y^{-1})) + \alpha \]
\[ = (\mu_A(x) \lor \mu_A(y)) + \alpha \]
\[ = (\mu_A(x) + \alpha) \lor (\mu_A(y) + \alpha) \]
\[ = \mu_T(x) \lor \mu_T(y). \]
Therefore, \( \mu_T(mxy^{-1}) \leq \mu_T(x) \lor \mu_T(y) \), for all \( x \) and \( y \) in \( G \).

And \[ v_T(mxy^{-1}) = v_A(mxy) + \beta \]
\[ \geq (v_A(x) \land v_A(y^{-1})) + \beta \]
\[ = (v_A(x) \lor v_A(y)) + \beta \]
\[ = (v_A(x) + \beta) \land (v_A(y) + \beta) \]
\[ = v_T(x) \land v_T(y). \]
Therefore, \( v_T(mxy^{-1}) \geq v_T(x) \land v_T(y) \), for all \( x \) and \( y \) in \( G \).

Hence \( T \) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \( G \).

2.4 Theorem: If \( T \) is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \( A \) of a M-group \( G \), then \( H = \{ x \in G : \mu_T(x) = \mu_T(e) \text{ and } v_T(x) = v_T(e) \} \) is a M-subgroup of \( G \).

Proof: Let \( x \), \( y \), and \( e \) be elements of \( G \).
Given \( H = \{ x \in G : \mu_T(x) = \mu_T(e) \text{ and } v_T(x) = v_T(e) \} \).

Now, \( \mu_T(x^{-1}) = \mu_T(x) \lor \mu_T(y) \)
\[ = \mu_T(e) \lor \mu_T(e) \]
\[ = \mu_T(e), \]
and \( \mu_T(e) = \mu_T((xy^{-1})(xy^{-1})^{-1}) \)
\[ = \mu_T(xy^{-1}) \lor \mu_T(xy^{-1}) \]
\[ = \mu_T(xy^{-1}). \]
Therefore, \( \mu_T(e) = \mu_T(xy^{-1}) \), for all \( x \) and \( y \) in \( G \).

Now, \[ v_T(xy^{-1}) \geq v_T(x) \land v_T(y) \]
\[ = v_T(e) \land v_T(e) \]
\[ = v_T(e), \]
and \[ v_T(e) = v_T((xy^{-1})(xy^{-1})^{-1}) \]
\[ = v_T(xy^{-1}) \lor v_T(xy^{-1}) \]
\[ = v_T(xy^{-1}). \]
Therefore, \( v_T(e) = v_T(xy^{-1}) \), for all \( x \) and \( y \) in \( G \).

Hence \( H \) is an M-subgroup of \( G \).

2.5 Theorem: Let \( T \) be an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \( A \) of a M-group \( G \). If \( \mu_T(xy^{-1}) = 0 \), then \( \mu_T(x) = \mu_T(y) \) and if \( v_T(xy^{-1}) = 1 \), then \( v_T(x) = v_T(y) \).

Proof: Let \( x \) and \( y \) be elements of \( G \).

Now, \[ \mu_T(x) = \mu_T(xy^{-1}y) \]
\[ \leq \mu_T(xy^{-1}) \lor \mu_T(y) \]
\[ = 0 \lor \mu_T(y) = \mu_T(y) \]
\[ = \mu_T(y) \]
\[ = \mu_T(x) \lor \mu_T(y) \]
\[ \leq \mu_T(x) \lor \mu_T(xy^{-1}) \]
\[ = \mu_T(x) \lor \mu_T(y) \].
Therefore, \( \mu_T(x) = \mu_T(y) \), for all \( x \) and \( y \) in \( G \).

Now, \[ v_T(x) = v_T(xy^{-1}y) \]
\[ \geq v_T(xy^{-1}) \land v_T(y) \]
\[ = 1 \land v_T(y) \]
\[ = v_T(y) \]
\[ = v_T(xy^{-1}) \]
\[ = v_T(xy^{-1}) \]
\[ \geq v_T(xy^{-1}) \land v_T(xy^{-1}) \]
\[ = v_T(x) \land v_T(y) \]
\[ = v_T(x) \land 1 = v_T(x). \]
Therefore, \( v_T(x) = v_T(y) \), for all \( x \) and \( y \) in \( G \).

2.6 Theorem: Let \( G \) be a M-group. If \( T \) is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \( A \) of \( G \), then \( \mu_T(xy) = \mu_T(x) \lor \mu_T(y) \) and \( v_T(xy) = v_T(x) \land v_T(y) \), for each \( x \) and \( y \) in \( G \) with \( \mu_T(x) \neq \mu_T(y) \) and \( v_T(x) \neq v_T(y) \).

Proof: Let \( x \) and \( y \) be elements of \( G \).
Assume that \( \mu_T(x) < \mu_T(y) \) and \( v_T(x) > v_T(y) \).

Then, \[ \mu_T(y) = \mu_T(xy) \]
\[ \leq \mu_T(x^{-1}) \lor \mu_T(xy) \]
\[ = \mu_T(x) \lor \mu_T(y) \]
\[ = \mu_T(y) \]
\[ = \mu_T(x) \lor \mu_T(y) \].
Therefore, \( \mu_T(xy) = \mu_T(y) = \mu_T(x) \lor \mu_T(y) \), for all \( x \) and \( y \) in \( G \).

Then, \[ v_T(y) = v_T(xy) \]
\[ \geq v_T(x^{-1}) \land v_T(xy) \]
\[ = v_T(xy) \land v_T(xy) \]
\[ = v_T(xy) \land v_T(y) \]
\[ = v_T(y). \]
Therefore, \( v_T(xy) = v_T(y) = v_T(x) \lor v_T(y) \), for all \( x \) and \( y \) in \( G \).

2.7 Theorem: Let \( (G, \ast) \) and \( (G', \ast') \) be any two M-groups. If \( f : G \rightarrow G' \) is a homomorphism, then the homomorphic image of an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \( A \) of a M-group \( G \) is an
proof: Let \((G, \cdot)\) and \((G', \cdot')\) be any two M-groups and \(f: G \rightarrow G'\) be a homomorphism. That is \(f(x y) = f(x) f(y), f(mx y) = mf(x) f(y)\), for all \(x\) and \(y\) in \(G\) and \(m\) in \(M\).

Let \(V = f(T^A_{(x, y)}), \) where \(T^A_{(x, y)}\) is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \(V\) of a M-group \(G'\).

We have to prove that \(V\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G'\).

Now, \(f(x)\) and \(f(y)\) in \(G'\), we have
\[
\mu_V\{ mf(x) ( f(y)^{-1} ) \} = \mu_V\{ mf(x) f(y)^{-1} \}
\]
\[
= \mu_V\{ f(mx y^{-1}) \}
\]
\[
\leq \mu^A\{ mx y^{-1}\}
\]
\[
= \mu_A\{ mx y^{-1}\} + \alpha
\]
\[
\leq \{ \mu_A(x) \lor \mu_A( y^{-1} ) \} + \alpha
\]
\[
\leq \{ \mu_A(x) \land \mu_A( y^{-1} ) \} + \alpha
\]
\[
= \{ \mu_A(x) + \alpha \} \lor \{ \mu_A( y^{-1} ) + \alpha \}
\]
\[
= \mu^A\{ x \} \lor \mu^A\{ y^{-1}\}
\]

which implies that
\[
\mu_V\{ mf(x) ( f(y)^{-1} ) \} \leq \mu_V\{ f(x) \} \lor \mu_V\{ f(y) \}, \) for all \(x\) and \(y\) in \(G'\).

And,
\[
v_V\{ mf(x) ( f(y)^{-1} ) \} = v_V\{ mf(x) f(y)^{-1} \}
\]
\[
= v_V\{ f(mx y^{-1}) \}
\]
\[
\geq v^A\{ mx y^{-1}\}
\]
\[
= v_A\{ mx y^{-1}\} + \beta
\]
\[
\geq \{ v_A(x) \land v_A( y^{-1} ) \} + \beta
\]
\[
\geq \{ v_A(x) \land v_A( y^{-1} ) \} + \beta
\]
\[
= (v_A(x) + \beta) \land (v_A( y^{-1} ) + \beta)
\]
\[
= v^A\{ x \} \land v^A\{ y^{-1}\}
\]

which implies that
\[
v_V\{ mf(x) f(y)^{-1} \} \geq v_V\{ f(x) \} \land v_V\{ f(y) \}, \) for all \(x\) and \(y\) in \(G'\).

Therefore, \(V\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G'\).

Hence the homomorphic image of an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G'\).

2.8 Theorem: Let \((G, \cdot)\) and \((G', \cdot')\) be any two M-groups. If \(f: G \rightarrow G'\) is a homomorphism, then the homomorphic pre-image of an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \(V\) of \(G'\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G\).

Proof: Let \((G, \cdot)\) and \((G', \cdot')\) be any two M-groups and \(f: G \rightarrow G'\) be a homomorphism. That is \(f(xy) = f(x)f(y), f(mx y) = mf(x) f(y),\) for all \(x\) and \(y\) in \(G\) and \(m\) in \(M\).

Let \(T = T^V_{(x, y)} = f(A),\) where \(T^V_{(x, y)}\) is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \(V\) of a M-group \(G'\).

We have to prove that \(A\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G\).

Let \(x\) and \(y\) in \(G\). Then,
\[
\mu_A\{ mx y^{-1} \} = \mu_T\{ f(mx y^{-1}) \}
\]
\[
= \mu_T\{ mf(x)f(y)^{-1} \}
\]
\[
= \mu_T\{ mf(x)( f(y)^{-1} ) \}
\]
\[
= \mu_T\{ mf(x)( f(y))^{1-1} \}
\]
\[
= \mu_T\{ mf(x)( f(y))^{-1} \} + \alpha
\]
\[
\leq \mu_V(f(x)) \lor \mu_V(f(y)) + \alpha
\]
\[
= \mu_V(f(x)) + \alpha \lor \mu_V(f(y)) + \alpha
\]
\[
= \mu_T(f(x)) \land \mu_T(f(y))
\]
\[
= \mu_A(x) \lor \mu_A(y)
\]

which implies that,
\[
\mu_A\{ mx y^{-1} \} \leq \mu_A(x) \lor \mu_A(y), \) for all \(x\) and \(y\) in \(G\).

And,
\[
v_A\{ mx y^{-1} \} = v_T\{ f(mx y^{-1}) \}
\]
\[
= v_T\{ mf(x)f(y)^{-1} \}
\]
\[
= v_T\{ mf(x)( f(y)^{-1} ) \}
\]
\[
= v_T\{ mf(x)( f(y))^{1-1} \}
\]
\[
= v_T\{ mf(x)( f(y))^{-1} \}
\]
\[
\geq \{ v_V(f(x)) \lor v_V(f(y)) \} + \beta
\]
\[
= (v_V(f(x)) + \beta) \land (v_V(f(y)) + \beta)
\]
\[
= v^A(x) \land v^A(y)
\]

which implies that,
\[
v_A\{ mx y^{-1} \} \geq v_A(x) \lor v_A(y), \) for all \(x\) and \(y\) in \(G\).

Therefore, \(A\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G\).

Hence the homomorphic pre-image of an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy M-subgroup \(V\) of a M-group \(G'\) is an intuitionistic anti L-fuzzy M-subgroup of a M-group \(G\).

2.9 Theorem: Let \((G, \cdot)\) and \((G', \cdot')\) be any two M-groups. If \(f: G \rightarrow G'\) is an anti-homomorphism, then the anti-homomorphic pre-image of an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy normal M-subgroup \(A\) of a M-group \(G\) is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group \(G'\).

Proof: Let \((G, \cdot)\) and \((G', \cdot')\) be any two M-groups and \(f: G \rightarrow G'\) be an anti-homomorphism.

That is \(f(xy) = f(x)f(y), f(mx y) = mf(x) f(y),\) for all \(x\) and \(y\) in \(G\) and \(m\) in \(M\).
Let $V = f(T^{A}_{(\alpha, \beta)})$, where $T^{A}_{(\alpha, \beta)}$ is an intuitionistic anti L-fuzzy translation of an intuitionistic anti L-fuzzy normal M-subgroup A of a M-group G.

We have to prove that V is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G$^i$. Now, for $f(x)$ and $f(y)$ in G$^i$, clearly V is an intuitionistic anti L-fuzzy M-subgroup of a M-group G$^i$. We have,

$$\mu_V( m f(x) f(y) ) = \mu_V( m f(myx) )$$

$$\leq \mu_V( myx )$$

$$= \mu_A(myx) + \alpha$$

$$= \mu_A(mxy) + \alpha$$

$$= \mu_I(mxy)$$

$$\leq \mu_V( f(mxy) )$$

$$= \mu_V( m f(y) f(x) )$$

which implies that,

$$\mu_V( m f(x) f(y) ) = \mu_V( m f(y) f(x) ), \text{ for } f(x) \text{ and } f(y) \text{ in } G$^i$.

And,

$$\nu_V( m f(x) f(y) ) = \nu_V( f(myx) )$$

$$\geq \nu_I(myx)$$

$$= \nu_A(mxy) + \alpha$$

$$= \nu_A(mxy) + \alpha$$

$$= \nu_I(mxy)$$

$$\geq \nu_V( f(mxy) )$$

$$= \nu_V( m f(y) f(x) )$$

which implies that,

$$\nu_V( m f(x) f(y) ) = \nu_V( m f(y) f(x) ), \text{ for } f(x) \text{ and } f(y) \text{ in } G$^i$.

Therefore, V is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G$^i$. Hence the anti-homomorphic image of an intuitionistic anti L-fuzzy translation of A of a M-group G is an intuitionistic anti L-fuzzy normal M-subgroup of a M-group G$^i$.

3. CONCLUSION

In this paper, we define a new algebraic structure of Intuitionistic anti L-fuzzy translation and Homomorphism and anti homomorphism of Intuitionistic anti L-fuzzy Translation, we wish to define Level subset of Intuitionistic anti L-fuzzy Translation and other some L-fuzzy Translation are in progress.

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