On Fuzzy Multi-Objective Multi-Item Solid Transportation Problems

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Abstract

In general. There is no single optimal solution in multi-objective problems, but rather a set of non inferior (pareto optimal) solutions from which the decision maker (DM) must select the most preferred or best compromise solution as the one to implement. In this paper, a multi-objective multi-item solid transportation problem by incorporating fuzzy numbers into the coefficients of the objective functions $(\tilde{c}_{i\,j\,k}^{r\,p})$ *and / or supply quantities* $(\tilde{a}_i^p)$ *and / or demand quantities* $(\tilde{b}_j^p)$ *and / or conveyance capacities* $(\tilde{e}_k)$*. The concept of* α*-fuzzy efficient solution is introduced in which the ordinary efficient solution is extended based on the* α*-levels of fuzzy numbers. A necessary and sufficient condition for such a solution is established. The existing results concerning the qualitative analysis of the notions (solvability set and stability set of the first kind under the concept of* α*-parametric optimality are studied. A solution procedure for determining the stability set of the first kind corresponding to one of* α*-pareto optimal solution is proposed. An illustrative numerical example is given to clarify the obtained results.*

Keywords: *Multi-objective multi-item solid transportation; fuzzy numbers;* α*-fuzzy efficient;* α*-optimality; parametric analysis.*

1. Introduction

The solid transportation problem (STP) is an extension of the classical transportation problem (TP) in which three item properties (supply, demand and conveyance) are taken into account in the constraint set instead of two items (supply and demand). The STP was first stated by Shell (1955) . A variety of approaches have been developed by many authors for multi-objective transportation problem (MOTP), Hussein (1998), and Das et al. (1999)). Some of the most important work related to STP are as follows:

Gen et al. (1995) proposed genetic algorithm for solving bicriteria STP with fuzzy numbers. Ojha et al. (2010) studied TP model with fixed charges and vechile costs where all unit discount (AUD). Incremental quantity discount (IQD) or combination of AUD and IQD on the price depending upon the amount is offered and varies on the choice of origin, destination and conveyance. Himenez and Verdegay (1999) introduced fuzzy STP in the case in which the fuzziness affects the constraint set. Ida et al. (1995) considered multi-criteria STP with fuzzy numbers; Li et al. (1997) presented a genetic algorithm for solving multi-objective STP with coefficient of the objective function as fuzzy numbers. Yang and Yuan (2007) investigated a bicriteria STP under stochastic environment. Bit et al. (1993) applied fuzzy programming approach to solve multi-objective STP. Nagarjan and Jeyaramon (2010) studied multi-objective STP with parameters as stochastic intervals. Kundu et al. (2013) introduced multi-objective multi-item STP with fuzzy coefficient for the objectives and constraints. Ammar and Khalifa (2014) studied multiobjective solid transportation problem with fuzzy numbers. Ammar and Khalifa (2015) studied multiobjective solid transportation problem with possibilistic variables.

Qualitative analysis of some basic notions such as the set of feasible parameters, solvability set, stability set of the first kind and stability set 0of the second and were introduced by Osman (1977). Sakawa and Yano (1989) introduced the concept of α-parametric optimality in fuzzy parametric programs.

In this paper, a multi-objective multi-item solid transportation problem (FMOMISTP) with fuzzy numbers in the objective functions coefficients, supply values, demand values and conveyance is studied. The concepts of fuzzy efficient and α-parametric efficient solutions are introduced. The relation between such solutions is given. A parametric analysis is used to characterize the set of all α-parametric efficient solutions. A solution procedure to determine the stability set of the first kind corresponding to one parametric efficient solution of FMOMISTP problem is presented. An illustrative numerical example is given in the sake of the paper to clarify the obtained results.

2. Preliminaries

Here, some definition needed through this paper are recalled.

Definition 1: (Dubois and Prade (1980)). A fuzzy number $(\tilde{q})$ is convex normalized fuzzy set of the real line *R* such that:

(a) $\exists\, x_0 \in R,\ \mu_{\tilde{q}}(x_0) = 1 (x_0$ is called the mean value of $\tilde{q}$);

(b) $\mu_{\tilde{q}}$ is piecewise continuous.

Definition 2: (Dubois and Prade (1980)). The α-level set of the fuzzy number $\tilde{q}$ is defined as the ordinary set $L_\alpha(\tilde{q})$ for which the degree of their membership functions exceed the level α:

$$(\tilde{q}) = \{q : \mu_{\tilde{q}}(q) \ge \alpha,\ \alpha \in [0, 1]\}.$$

Throughout this paper, $F(R)$ denoted the set of all compact (i.e., bounded and closed) fuzzy numbers on *R*, that is, for an $f \in F(R)$ of satisfies:

(i) There exists $\alpha \in R$ such that $f(x) = 1$;

(ii) For any $\alpha \in (0, 1],\ (\tilde{f})_\alpha = [f_\alpha^L, f_\alpha^U]$ is closed interval on *R*.

Observe that $R \subset F(R)$.

The following notations are used in FMOMISTP: $\tilde{c}_{ijk}^{rp}$, $\tilde{a}_i^p$, $\tilde{b}_j^p$ and $\tilde{e}_k \in F(R)$, $r = 1, ..., s;\ p = 1, ..., t;\ i = 1, ..., m;\ j = 1, ..., n;\ k = 1, ..., \ell$, and their α-level sets are:

$$(\tilde{c}_{ijk}^{rp})_\alpha = (\tilde{c})_\alpha = \{c \in R^{s(t*m*n*\ell)} : \mu_{\tilde{c}_{ijk}^{rp}}(c_{ijk}^{rp}) \ge \alpha,\ r = 1, ..., s;\ p = 1, ..., t;$$

$$i 1, ..., m;\ j = 1, ...,\ n;\ k = 1, ..., \ell\},$$

$$(\tilde{a}_i^p)_\alpha = (\tilde{a})_\alpha = \{a \in R^{t*m} : \mu_{\tilde{a}_i^p}(a_i^p) \ge \alpha,\ p = 1, ..., t;\ i = 1, ..., m\},$$

$$(\tilde{b}_j^p)_\alpha = (\tilde{b})_\alpha = \{b \in R^{t*n} : \mu_{\tilde{b}_j^p}(b_j^p) \ge \alpha,\ p = 1, ..., t;\ j = 1, ..., n\},$$

$$(\tilde{e}_k)_\alpha = (\tilde{e})_\alpha = \{e \in R^\ell : \mu_{\tilde{e}_k}(e_k) \ge \alpha,\ k = 1, ..., \ell\},$$

where $\tilde{c}_{ijk}^{rp}$, $\tilde{a}_i^p$, $\tilde{b}_j^b$, $\tilde{e}_k$ are denoted as fuzzy objective functions coefficients, fuzzy supplies, fuzzy demands and fuzzy conveyance, respectively.

3. Problem formulation

A multi-objective multi-item solid transportation problem with fuzzy parameters (FMOMSTP) is

(FMOMSTP) $$\min z_r(x, \tilde{c}^r) = \sum_{p=1}^{t}\sum_{i=1}^{m}\sum_{p=1}^{n}\sum_{k=1}^{\ell} \tilde{c}_{ijk}^{rp} x_{ijk}^{p}, \qquad r = 1, \dots, s$$

subject to

$$M(\tilde{a}, \tilde{b}, \tilde{e}) = \{x \in R^{t*m*n*\ell} : \sum_{j=1}^{n}\sum_{k=1}^{\ell} x_{ijk}^{p} \le \tilde{a}_i^p, \ p = 1, \dots, t;\ i = 1, \dots, m;$$

$$\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk}^{p} \ge \tilde{b}_j^p, \qquad p = 1, \dots, t;\ j = 1, \dots, n;$$

$$\sum_{p=1}^{t}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \le \tilde{e}_k, \qquad k = 1, \dots, \ell;$$

$$\sum_{i=1}^{m} \tilde{a}_i^p \ge \sum_{j=1}^{n} \tilde{b}_j^p, \qquad p = 1, \dots, t,$$

$$\sum_{k=1}^{\ell} \tilde{e}_k \ge \sum_{p=1}^{t}\sum_{j=1}^{n} \tilde{b}_j^p,$$

$$x_{ijk}^{p} \ge 0, \qquad p = 1, \dots, t;\ \ i = 1, \dots, m;\ j = 1, \dots, n;\ k = 1, \dots, \ell\}$$

where $p(=1, \dots, t)$ items are to be transported from m origins to n distributions by means of k $(=1, \dots, \ell)$ different modes of transportation (conveyance). For the objective z_r, $\tilde{c}_{ijk}^{rp}$ represents fuzzy unit transportation penalty from i^{th} origin to j^{th} destination by k^{th} conveyance for p^{th} item. $\tilde{a}_i^p$ and $\tilde{b}_j^p$ represent total fuzzy supply of i^{th} origin and total fuzzy demand of j^{th} destination, respectively for p^{th} item. Also, $\tilde{e}_k$ is the total fuzzy capacity of k^{th} conveyance. All of $\tilde{c}_{ijk}^{rp}$, $\tilde{a}_i^p$, $\tilde{b}_j^p$, and $\tilde{e}_k$ are assumed to be characterized as trapezoidal fuzzy numbers, and $M(\tilde{a}, \tilde{b}, \tilde{e})$ is compact set.

Definition 3. (*α*-fuzzy Feasible actions): Let $\alpha_1 = (\alpha_{11}, \dots, \alpha_{1m})$ $\alpha_{ii} \in [0, 1]$, $i = 1, \dots, m$; $\alpha_2 = (\alpha_{21}, \dots, \alpha_{2n})$, $\alpha_{2j} \in [0, 1]$, $j = 1, \dots, n$; and $\alpha_3 = (\alpha_{31}, \dots, \alpha_{3n})$, $\alpha_{3k} \in [0, 1]$, $k = 1, \dots, \ell$;. Then

$x \in M = \{x \in R^{t*m*n*l} : x_{ijk}^{p} \ge 0, p = 1,\dots,t; i = 1,\dots,m; j = 1,\dots,n; k = 1,\dots,l\}$ is said to be

α -possible actions for problem (FMOMISTP) if :

$$\mu\left(\sum_{j=1}^{n}\sum_{k=1}^{\ell} x_{ijk}^{p} \le \tilde{a}_i^p\right) = \sup_{\sum_{j=1}^{n}\sum_{k=1}^{\ell} x_{ijk}^{r} \le \tilde{a}_i^p} (\mu_{\tilde{a}_i^p}) \ge \alpha_{1i}, \qquad i = 1, \dots, m;$$

$$\mu\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk}^{p} \geq \tilde{b}_{j}^{p}\right) = \sup_{\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk}^{r} \geq \tilde{b}_{j}^{p}} (\mu_{\tilde{b}_{j}^{p}}) \geq \alpha_{2j}, \qquad j = 1, \ldots, n;$$

$$\mu\left(\sum_{p=1}^{t}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \leq \tilde{e}_{k}\right) = \sup_{\sum_{p=1}^{t}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{p} \leq \tilde{e}_{k}} (\mu_{\tilde{e}_{k}}) \geq \alpha_{3k}, \qquad k = 1, \ldots, \ell;$$

where μ denotes membership function.

Definition 4. (α-fuzzy efficient): A point $x^{*}(\tilde{c}^{*}) \in M(\tilde{a}^{*}, \tilde{b}^{*}, \tilde{e}^{*})$ is said to α-fuzzy efficient solution to FMOMISTP problem if there is no $x(\tilde{c}^{*}) \in G(\tilde{a}^{*}, \tilde{b}^{*}, \tilde{e}^{*})$ such that

$$\begin{aligned}\mu_{\tilde{c}}\{c \in R^{s(t*m*n*\ell)} &: z_1(x, \tilde{c}^{r1}) \leq z_1(x^{*}, \tilde{c}^{*1}), \ldots, z_{r-1}(x, \tilde{c}^{(r-1)} \leq \\ & z_{r-1}(x^{*}, \tilde{c}^{(r-1)}), z_r(x, \tilde{c}^{r}) < z_r(x^{*}, \tilde{c}^{*r}), z_{r+1}(x, \tilde{c}^{(r+1)}) \leq \\ & z_{r+1}(x^{*}, \tilde{c}^{(r+1)}), \ldots z_s(x, \tilde{c}^{*s}) \leq z_s(x^{*}, \tilde{c}^{*s})\} \geq \alpha, \ \alpha \in [0, 1]\end{aligned} \qquad (1)$$

where μ denotes membership function. On account of the extension principle,

$$\begin{aligned}\mu_{\tilde{c}}\{c \in R^{s(t*m*n*\ell)} &: z_1(x, \tilde{c}^{*1}) \leq z_1(x^{*}, \tilde{c}^{*1}), \ldots, z_{r-1}(x, \tilde{c}^{(r-1)} \leq \\ & z_{r-1}(x^{*}, \tilde{c}^{(r-1)}), z_r(x, \tilde{c}^{*r}) < z_r(x^{*}, \tilde{c}^{*r}), z_{r+1}(x, \tilde{c}^{(r+1)}) \leq \\ & z_{r+1}(x^{*}, \tilde{c}^{(r+1)}), \ldots, z_s(x, \tilde{c}^{*s}) \leq z_s(x^{*}, \tilde{c}^{*s})\} = \\ & \sup_{(c^1, \ldots, c^s) \in c} \min(\mu_{\tilde{c}^1}(c^1), \ldots, \mu_{\tilde{c}^s}(c^s)),\end{aligned} \qquad (2)$$

where

$$\begin{aligned}c = \{c^1, \ldots, c^s) \in R^{s(t*m*n*\ell)} &: z_1(x, c^{*1}) \leq z_1(x^{*}, \tilde{c}^{*1}), \ldots, z_{r-1}(x, \tilde{c}^{*(r-1)}) \leq \\ & z_{r-1}(x^{*}, c^{*(r-1)}), \ldots, z_s(x, \tilde{c}^{*s}) \leq z_s(x^{*}, \tilde{c}^{*s})\}\end{aligned} \qquad (3)$$

and $\mu_{\tilde{c}_{ijk}^{rp}}$ $(r = 1, \ldots, s;\ p = 1, \ldots, t;\ i = 1, \ldots, m;\ j = 1, \ldots, n;\ k = 1, \ldots, \ell)$ are $s(t*m*n*\ell)$ any membership functions.

4. Characterizing α-fuzzy efficient solution for FMOMISTP problem

For characterizing the α - fuzzy efficient solution for FMOMISTP problem, let us consider the following α-parametric multi-objective multi-item solid transportation problem (α-PMOMISTP)

$$(\alpha\text{-PMOMISTP}) \qquad \min z_r(x, \tilde{c}) = \sum_{p=1}^{t}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} \tilde{c}_{ijk}^{p}, \quad r = 1, \ldots, s$$

subject to

$$x \in M(a, b, e), \ c_{ijk}^{rp} \in (\tilde{c}_{ijk}^{rp})_{\alpha}, \ r = 1, \ldots, s; \ p = 1, \ldots, t; \ i = 1, \ldots, m;$$

$$j = 1, \ldots, n;\ k = 1, \ldots, \ell;\ a_i \in (\tilde{a}_i^p)_\alpha,\ i = 1, \ldots, m;\ p = 1, \ldots, t;$$

$$b_j \in (\tilde{b}_j^p)_\alpha,\ j = 1, \ldots, n;\ p = 1, \ldots, s;\ e_k \in (\tilde{e}_k)_\alpha,\ k = 1, \ldots, \ell;$$

$$\sum_{i=1}^{m} (\tilde{a}_i^p)_\alpha \geq \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha,\ p = 1, \ldots, t;\ \sum_{k=1}^{\ell} (\tilde{e}_k^p)_\alpha \geq \sum_{p=1}^{t} \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha, \text{ and}$$

$$x_{i\,j\,k}^p \geq 0,\ p = 1, \ldots, t;\ i = 1, \ldots, m;\ j = 1, \ldots, n;\ k = 1, \ldots, \ell,$$

where $(\tilde{c}_{i\,j\,k}^{r\,p})_\alpha$, $(\tilde{a}_i^p)_\alpha$, $(\tilde{b}_j^p)_\alpha$, and $(\tilde{e}_k)_\alpha$ denote the α-cuts of the fuzzy variables $c_{i\,j\,k}^{r\,p}$, a_i^p, b_j^p, and e_k, respectively. By the convexity assumption, $\mu_{\tilde{c}_{i\,j\,k}^{r\,p}}(c_{i\,j\,k}^{r\,p})$, $(\tilde{c}_{i\,j\,k}^{r\,p})_\alpha$; $\mu_{\tilde{a}_i^p}(a_i^p)$, $(\tilde{a}_i^p)_\alpha$; $\mu_{\tilde{b}_j^p}(b_j^p)$, $(\tilde{b}_j^p)_\alpha$; and $\mu_{\tilde{e}_k}(e_k)$, $(\tilde{e}_k)_\alpha$, $r = 1, \ldots, s$; $p = 1, \ldots, t$; $i = 1, \ldots, m$; $j = 1, \ldots, n$; $k = 1, \ldots, \ell$ are real intervals that will denoted as: $[(c_{i\,j\,k}^{r\,p}(\alpha))^L, (c_{i\,j\,k}^{r\,p}(\alpha))^U]$, $[(a_i^p(\alpha))^L, (a_i^p(\alpha))^U]$, $[(b_j^p(\alpha))^L, (b_j^p(\alpha))^U]$ and $[(e_k(\alpha))^L,\ (e_k(\alpha))^U]$, respectively.

Definition 5. (α-parametric efficient): A point $x^*(c^*) \in M(a^*, b^*, e^*)$ is said to be α-parametric efficient solution for α-PMOMISTP problem if there are no $x \in M(a^*, b^*, e^*)$ is said to be α-parametric efficient solution for α-PMOMISTP problem if there are no $x \in M(a^*, b^*, e^*)$, $c_{i\,j\,k}^{r\,p} \in [(c_{i\,j\,k}^{r\,p}(\alpha))^L,\ (c_{i\,j\,k}^{r\,p}(\alpha))^U]$, $a_i^p \in [\,(a_i^p(\alpha))^L,\ (a_i^p(\alpha))^U\,]$, $b_j^p \in [(b_j^p(\alpha))^L,\ (b_i^p(\alpha))^U\ ]$ and

$e_k \in [(e_k(\alpha))^L, (e_k(\alpha))^U]$ such that: $z_r(x, c^r) \leq z_r(x^*, c^r)$, for all $r = 1, \ldots, s$ and strict inequality holds for at least one *r*.

Theorem 1. $x^* \in M(\tilde{a}^*, \tilde{b}^*, \tilde{e}^*)$ is α-fuzzy efficient solution for FMOMUSTP problem if and only if $x^* \in M(a^*, b^*, e^*)$ is $x^* \in M(\tilde{a}^*, \tilde{b}^*, \tilde{e}^*)$ parametric efficient solution for α-PMOMISTP problem.

Proof: (The proof will be in contra positive direction).

Necessity: Let $x^*(\tilde{c}^*) \in M(\tilde{a}^*, \tilde{b}^*, \tilde{e}^*)$ be α-fuzzy efficient solution for FMOMISTP problem and $x^*(c^*) \in M(a^*, b^*, e^*)$ be not α-parametric efficient solution for α-PMOMISTP problem. Then there is $x^1(\tilde{c}^*) \in M(\tilde{a}^*, \tilde{b}^*, \tilde{e}^*)$ for $c^* \in (\tilde{c}^*)_\alpha$, $a^* \in (\tilde{a}^*)_\alpha$, $b^* \in (\tilde{b}^*)_\alpha$, and $e^* \in (\tilde{e}^*)_\alpha$, such that:

$$z_r(x^1, \tilde{c}^r) \le z_r(x^*, \tilde{c}^r),\ r = 1, \dots, s,$$

with strict inequality holds for at least one. This leads to:

$$\mu_{\tilde{c}}\{c \in R^{s(t*m*n*\ell)} : z_1(x, \tilde{c}^{*1}) \le z_1(x^*, \tilde{c}^{*1}), \dots, z_{r-1}(x^1, \tilde{c}^{*(r-1)}) \le z_{r-1}(x^*, \tilde{c}^{*(r-1)}), z_r(x^1, \tilde{c}^{*r}) < z_r(x^*, \tilde{c}^{*r}), z_{r+1}(x^1, \tilde{c}^{*(r+1)}) \le z_{r+1}(x^*, \tilde{c}^{*(r+1)}), \dots, z_s(x^1, \tilde{c}^{*s}) \le z_s(x^*, \tilde{c}^{*s})\} \ge \alpha.$$

This contradicts the α-fuzzy efficient of $x^*(\tilde{c}^*) \in M(\tilde{a}^*, \tilde{b}^*, \tilde{e}^*)$ for FMOMISTP problem and the necessary part is established.

Sufficiency: Let $x^*(c^*) \in M(a^*, b^*, e^*)$ be α-parametric efficient solution for α-PMOMISTP problem and $x^*(\tilde{c}^*) \in M(\tilde{a}^*, \tilde{b}^*, \tilde{e}^*)$ be not α-fuzzy efficient solution for FMOMISTP problem. Then there is $x^2(c^*) \in M(a^*, b^*, e^*)$ such that:

$$\mu_{\tilde{c}}\{c \in R^{s(t*m*n*\ell)} : z_1(x^2, c^{*1}) \le z_1(x^*, c^{*1}), \dots, z_{r-1}(x^2, c^{*(r-1)}) \le z_{r-1}(x^*, c^{*(r-1)}), z_r(x^2, c^{*r}) < z_r(x^*, c^{*r}), z_{r+1}(x^2, c^{*(r+1)}) \le z_{r+1}(x^*, c^{*(r+1)}), \dots, z_s(x^2, c^{*s}) \le z_s(x^*, c^{*s})\} \ge \alpha,$$

i.e.,

$$\sup_{(c^1, \dots, c^s) \in \bar{c}} \min(\mu_{\tilde{c}^1}(c^1), \dots, \mu_{\tilde{c}^s}(c^s)) \ge \alpha, \qquad (4)$$

where

$$\bar{c} = \{(c^1, \dots, c^s) \in R^{s(t*m*n*\ell)} : z_1(x^2, c^{*1}) \le z_1(x^*, c^{*1}), \dots, z_{r-1}(x^2, c^{*(r-1)}) \le z_{r-1}(x^*, c^{*r}), z_r(x^2, c^{*r}) < z_r(x^*, c^{*r}), \dots, z_{r+1}(x^2, c^{*(r+1)}) \le z_{r+1}(x^2, c^{*s}) \le z_s(x^*, c^{*s})\}.$$

For this supremum to exist, there is $(d^1, \dots, d^s) \in \bar{c}$ with $\min(\mu_{\tilde{c}^1}(d^1), \dots, \mu_{\tilde{c}^s}(d^s) < \alpha$, then

$$\sup_{(d^1, \dots, d^s) \in \bar{c}} \min(\mu_{\tilde{c}^1}(d^1), \dots, \mu_{\tilde{c}^s}(d^s)) < \alpha.$$

This contradicts (4). Then there is $(d^1, \dots, d^s) \in \bar{c}$ satisfying

$$\min(\mu_{\tilde{c}^1}(d^1), \dots, \mu_{\tilde{c}^s}(d^s)) \ge \alpha \qquad (5)$$

This contradicts the efficient of $x^* \in M(a^*, b^*, e^*)$ for α-PMOMISTP problem, and the sufficiency part is proved.

5. Parametric analysis

For characterizing the set of all parametric efficient solutions for α-PMOMISTP problem, we use the parametric optimization (scalarization) problem (here the weighting method (Chanas et al. (1984)).

STP (w) $\quad \min \sum_{r=1}^{s} w_r z_r(x, c^r)$

subject to

$$x \in M(a, b, e),\ c_{ijk}^{rp} \in (\tilde{c}_{ijk}^{rp})_\alpha,$$

$$a_i^p \in (\tilde{a}_i^p)_\alpha,\ b_j^p \in (\tilde{b}_j^p)_\alpha,\ e_k \in (\tilde{e}_k)_\alpha,$$

$$\sum_{i=1}^{m} (\tilde{a}_i^p)_\alpha \geq \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha,$$

$$\sum_{k=1}^{\ell} (\tilde{e}_i^p)_\alpha \geq \sum_{p=1}^{t} \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha,$$

$$w_r > 0,\ r = 1, \ldots, s,\ \sum_{r=1}^{s} w_r = 1, \text{ and}$$

$$x_{ijk}^p \geq 0,\ p = 1, \ldots, t;\ i = 1, \ldots, m;\ j = 1, \ldots, n;\ k = 1, \ldots, \ell.$$

Definition 6. (α-parametric optimal): The set of α-parametric optimal solutions of STP(w) is defined as:

$$E(w) = \left\{ x^* \in R^{t*m*n*\ell} : \sum_{r=1}^{s} w_r z_r(x^*, c^{r*}) \leq \sum_{r=1}^{s} w_r z_r(\alpha, c^{r*}), \right.$$

$$\text{for each } x(c^*) \in M(a^*, b^*, e^*), c^* \in (\tilde{c})_\alpha,\ a^* \in (\tilde{a})_\alpha,$$

$$\left. b^* \in (\tilde{b})_\alpha,\ e^* \in (\tilde{e})_\alpha, \text{ and } w_r > 0,\ r = 1, \ldots, s,\ \sum_{r=1}^{s} w_r = 1 \right\}.$$

Remark 1. A point $x^*(c^*)$ is said to be α-parametric efficient solution of α-PMOMISTP problem with corresponding α-parametric optimal parameters (c^*, a^*, b^*, e^*) if there exists $w^* > 0$ such that x^* is the unique α-parametric optimal solution of STP(w).

Remark 2. A point $x^*(c^*)$ is said to be a proper α-parametric efficient solution of α-PMOMISTP problem if and only if there exists $w^* > 0$ such that $x^* \in E(w^*)$.

Definition 7. The solvability set of α-PMOMISTP problem is denoted by B and is defined by:

$$B = \{w \in R^s : \text{there exists } \alpha\text{-parametric efficient solution } x^* \text{ of } \alpha\text{-PMOMISTP problem}, x^* \in E(w)\}.$$

Definition 8. The proper solvability set of α-PMOMISTP problem is defined as:

$$B' = \{w \in B,\ w > 0\}.$$

Remark 3. If $B' \subset B$ and $x^* \in E(w^*)$, $w^* \in B'$, then x^* is called a proper α-parametric efficient solution of α-PMOMISTP problem.

Definition 9. Suppose that $x^* \in B$ with a corresponding α-parametric optimal solution $x^* \in E(w^*)$, then the stability set of the first kind of α-PMOMISTP problem corresponding to x^*, denoted as $S(x^*)$ and is defined by:

$$S(x^*) = \{w \in B : x^* \in E(w^*) \text{ is an } \alpha\text{-parametric optimal solution of STP}(w) \text{ problem}\}.$$

For determining the stability set of the first kind, let STP(w) reformulated as in the following form:

$$(\text{STP}(w))' \qquad \min \sum_{r=1}^{s} w_r z_r(x, c^r)$$

subject to

$$f_i(x, a_i^p) = \left(= \sum_{j=1}^{n} \sum_{k=1}^{\ell} x_{ijk}^p \le a_i^p,\ p = 1, \ldots, t;\ i = 1, \ldots, m \right);$$

$$g_i(x, b_j^p) = \left(= \sum_{i=1}^{m} \sum_{k=1}^{\ell} x_{ijk}^p \ge b_j^p,\ p = 1, \ldots, t;\ j = 1, \ldots, n \right);$$

$$h_k(x, e_k) = \left(= \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p \le e_k,\ p = 1, \ldots, t;\ k = 1, \ldots, \ell \right);$$

$$\mu_{\tilde{c}_{ijk}^{rp}}(c_{ijk}^{rp}) \ge \alpha,\ r = 1, \ldots, s;\ p = 1, \ldots, t;\ i = 1, \ldots, m;$$
$$j = 1, \ldots, n;\ k = 1, \ldots, \ell;$$

$$\mu_{\tilde{a}_i^p}(a_i^p) \ge \alpha,\ p = 1, \ldots, t;\ i = 1, \ldots, m;$$

$$\mu_{\tilde{b}_j^p}(b_j^p) \ge \alpha,\ p = 1, \ldots, t;\ j = 1, \ldots, n;$$

$$\mu_{\tilde{e}_k}(e_k) \ge \alpha,\ k = 1, \ldots, \ell;$$

$$\sum_{i=1}^{m} (\tilde{a}_i^p)_\alpha \ge \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha,\ p = 1, \ldots, t;$$

$$\sum_{k=1}^{\ell} (\tilde{e}_k)_\alpha \ge \sum_{p=1}^{r} \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha;$$

$$w_r > 0,\ r = 1, \ldots, s;\ \sum_{r=1}^{s} w_r = 1,$$

$$x_{i\,j\,k}^{\,p} \geq 0,\; p=1,\ldots,t;\; i=1,\ldots,m;\; j=1,\ldots,n;\; k=1,\ldots,\ell$$

and $\alpha \in [0,1]$.

Let $w^{*} \in B$ with x^{*} is α-parametric efficient solution for α-PMOMISTP problem, then the Kuhn-Tucker necessary optimality conditions corresponding to (STP $(w))'$ take the form:

$$\sum_{r=1}^{s} w_r \frac{\partial z_r}{\partial x_\eta} - \sum_{i=1}^{m} \delta_i \frac{\partial f_i}{\partial x_\eta} + \sum_{j=1}^{n} \gamma_j \frac{\partial g_i}{\partial x_\eta} + \sum_{k=1}^{\ell} \xi_k \frac{\partial h_k}{\partial x_\eta} = 0,\; \eta = 1,\ldots,t*m*n*\ell;$$

$$\sum_{r=1}^{s} w_r \frac{\partial z_r}{\partial c_{i\,j\,k}^{r\,p}} - \sum_{r=1}^{s} u_r \frac{\partial \mu_{\tilde{c}_{i\,j\,k}}^{r\,p}(c_{i\,j\,k}^{r\,p})}{\partial c_{i\,j\,k}^{r\,p}} = 0,\; r=1,\ldots,s;\; p=1,\ldots,t;\; i=1,\ldots,m;$$

$$j=1,\ldots,m;\; k=1,\ldots,\ell;$$

$$\sum_{i=1}^{m} \delta_i \frac{\partial f_i}{\partial a_i^p} - \sum_{i=1}^{m} \phi_i \frac{\partial \mu_{\tilde{a}_i^p}(a_i^p)}{\partial a_i^p} = 0, \quad p=1,\ldots,t;$$

$$\sum_{i=1}^{m} \delta_i \frac{\partial f_i}{\partial a_i^p} - \sum_{i=1}^{m} \phi_i \frac{\partial \mu_{\tilde{a}_i^p}(a_i^p)}{\partial a_i^p} = 0, \quad p=1,\ldots,t;$$

$$\sum_{j=1}^{n} \gamma_j \frac{\partial g_i}{\partial b_j^p} - \sum_{j=1}^{n} \psi_j \frac{\partial \mu_{\tilde{b}_j^p}(b_j^p)}{\partial b_j^p} = 0,\quad p=1,\ldots,t;$$

$$\sum_{k=1}^{\ell} \xi_k \frac{\partial h_k}{\partial e_k} - \sum_{k=1}^{\ell} \rho_k \frac{\partial \mu_{\tilde{e}_k}(e_k)}{\partial e_k} = 0,$$

$$f_i(x, a_i^p)\left(= \sum_{j=1}^{n}\sum_{k=1}^{\ell} x_{i\,j\,k}^{\,p} \leq a_i^p,\; p=1,\ldots,t;\; i=1,\ldots,m \right);$$

$$g_j(x, b_j^p)\left(= \sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{i\,j\,k}^{\,p} \geq b_j^p,\; p=1,\ldots,t;\; j=1,\ldots,n \right);$$

$$h_u(x, e_k)\left(= \sum_{i=1}^{m}\sum_{j=1}^{n} x_{i\,j\,k}^{\,p} \leq e_k,\; p=1,\ldots,t;\; k=1,\ldots,\ell \right);$$

$$\mu_{\tilde{c}_{i\,j\,k}^{r\,p}}(\tilde{c}_{i\,j\,k}^{r\,p}) \geq \alpha, \qquad r=1,\ldots,s;\; p=1,\ldots,t;$$

$$\mu_{\tilde{c}_{i\,j\,k}^{r\,p}}(\tilde{c}_{i\,j\,k}^{r\,p}) \geq \alpha, \qquad r=1,\ldots,s;\; p=1,\ldots,t;\; i=1,\ldots,m;$$

$$j=1,\ldots,m;\; k=1,\ldots,\ell;$$

$$\mu_{\tilde{a}_i^p}(a_i^p) \geq \alpha, \qquad p=1,\ldots,t;\; i=1,\ldots,m;$$

$$\mu_{\tilde{b}_j^p}(b_j^p) \geq \alpha, \qquad p=1,\ldots,t;\; j=1,\ldots,n;$$

$$\mu_{\tilde{e}_k}(e_k) \geq \alpha, \qquad k = 1, \ldots, \ell;$$

$$\sum_{i=1}^{m} (\tilde{a}_i^p)_\alpha \geq \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha, \qquad p = 1, \ldots, t;$$

$$\sum_{k=1}^{\ell} (\tilde{e}_k)_\alpha \geq \sum_{p=1}^{t} \sum_{j=1}^{n} (\tilde{b}_j^p)_\alpha,$$

$$\delta_i f_i(x, a_i^p) = 0, \qquad p = 1, \ldots, t;\ i = 1, \ldots, m;$$

$$\gamma_j g_j(x, b_j^p) = 0, \qquad p = 1, \ldots, t;\ j = 1, \ldots, n;$$

$$\xi_k h_k(x, e_k) = 0, \qquad k = 1, \ldots, \ell;$$

$$U_r(-\mu_{\tilde{c}_{i\,j\,k}^{r\,p}}(\tilde{c}_{i\,j\,k}^{r\,p}) + \alpha) = 0, \quad r = 1, \ldots, s; e = 1, \ldots, t; i = 1, \ldots, m; j = 1, \ldots, n;$$

$$k = 1, \ldots, \ell;$$

$$\phi_i(-\mu_{\tilde{a}_i^p}(a_i^p) + \alpha) = 0, \qquad p = 1, \ldots, t;\ i = 1, \ldots, m;$$

$$\psi_j(-\mu_{\tilde{b}_j^p}(b_j^p) + \alpha) = 0, \qquad p = 1, \ldots, t;\ j = 1, \ldots, n;$$

$$\rho_k(-\mu_{\tilde{e}_k}(e_k) + \alpha) = 0, \qquad k = 1, \ldots, \ell;$$

$$\delta_i,\ \gamma_j,\ \xi_k,\ U_r,\ \phi_i,\ \psi_j \text{ and } \rho_k \geq 0,$$

$$i = 1, \ldots, m;\ j = 1, \ldots, n;\ k = 1, \ldots, \ell,$$

where δ_i, γ_j, ξ_k, U_r, ϕ_i, ψ_j and ρ_k are the Lagrange multipliers and the above expressions are evaluated at $(x^*, c^*, a^*, b^*, e^*)$. The first four relations with the last one of the above system represent a pobytope *T* for which vertices can be determined using any algorithm based on the simplex method. According to whether any of the variables $\delta_i\ (i = 1, \ldots, m)$, $\gamma_j\ (j = 1, \ldots, n)$, $\xi_k\ (k = 1, \ldots, \ell)$, $U_r\ (r = 1, \ldots, s)$, ϕ_i, ψ_j and ρ_k are zero or positive, the stability set of the first kind of (STP $(w))'$ is determined.

6. Solution procedure

The steps of the proposed procedure to determine $S(x^*)$ can be summarized as in the following steps:

Step 1: Ask the DM to specify the initial value of $\alpha\,(0 \leq \alpha \leq 1)$, say $\alpha^* = 0$).

Step 2: Elicit a membership function for each of the fuzzy numbers $\tilde{c}_{i\,j\,k}^{r\,p}$, $\tilde{a}_i^p$, $\tilde{b}_j^p$, and $\tilde{e}_k$, $r = 1, \ldots, s;\ p = 1, \ldots, t;\ i = 1, \ldots, m,\ j = 1, \ldots, n;\ k = 1, \ldots, \ell$ in α-PMOMISTP problem.

Step 3: Construct the parametric nonlinear programming problem (STP $(w))'$.

Step 4: Choose certain $w^* \in B$ and solve (STP $(w))'$ problem using any available computer package. Let x^* be α-parametric optimal solution of (STP $(w))'$ with the corresponding α-level optimal parameters (c^*, a^*, b^*, e^*).

Step 5: Substitute with x^*, c^*, a^*, b^* and e^* in the Kuhn-Tucker necessary optimality conditions and hence solve the resulted system.

Step 6: Determine $S(x^*)$ according to the values of the Lagrange multipliers.

Step 7: Set $\alpha^1 = (\alpha^* + \varepsilon) \in (0, 1]_0$ and go to step 2.

Repeat the above steps of the procedure until the interval $[0, 1]$ is fully exhausted, they stop.

7. Numerical example

Consider the FMOMISTP problem with $p = 1, 2 = i, j, k, r$. The unit transportation are given in the following tables:

Table 1. Fuzzy unit transportation penalties / costs $\tilde{c}^{11}_{ijk}$

i	*j*		*j*	
	1	**2**	**1**	**2**
1	(6, 8, 9, 11)	(4, 6, 9, 11)	(9, 11, 13, 15)	(6, 8, 10, 12)
2	(8, 10, 13, 15)	(6, 7, 8, 9)	(10, 12, 13, 15)	(6, 8, 10, 12)
k	1		2	

Table 2. Fuzzy unit transportation penalties / costs $\tilde{c}^{12}_{ijk}$

i	*j*		*j*	
	1	**2**	**1**	**2**
1	(9, 10, 12, 13)	(6, 8, 10, 12)	(11, 13, 14, 16)	(6, 7, 10, 11)
2	(11, 13, 14, 16)	(7, 9, 12, 14)	(14, 16, 18, 20)	(9, 11, 12, 14)
k	1		2	

Table 3. Fuzzy unit transportation penalties / costs $\tilde{c}^{21}_{ijk}$

i	*j*		*J*	
	1	**2**	**1**	**2**

1	(4, 5, 7, 8)	(3, 5, 6, 8)	(6, 7, 8, 9)	(4, 6, 7, 9)
2	(6, 8, 9, 11)	(4, 6, 8, 10)	(4, 6, 8, 10)	(7, 9, 11, 13)
k	1		2	

Table 4. Fuzzy unit transportation penalties / costs $\tilde{c}_{i\,j\,k}^{22}$

i	***j***		***J***	
	1	**2**	**1**	**2**
1	(5, 7, 8, 10)	(4, 6, 7, 9)	(7, 8, 9, 10)	(4, 6, 7, 9)
2	(10, 11, 13, 14)	(6, 7, 8, 9)	(6, 8, 10, 12)	(5, 7, 9, 11)
k	1		2	

The fuzzy supply quantities $\tilde{a}_i^p$, $i = p = 1, 2$, fuzzy demand quantities $\tilde{b}_j^p$, $p = j = 1, 2$ and fuzzy conveyances $\tilde{e}_k$, $k = 1, 2$, are:

Table 5. Fuzzy supply quantities $\tilde{a}_i^p$

$\tilde{a}_1^1 = (22, 24, 26, 28)$	$\tilde{a}_1^2 = (32, 34, 37, 39)$
$\tilde{a}_2^1 = (30, 32, 35, 37)$	$\tilde{a}_2^2 = (25, 28, 30, 33)$

Table 6. Fuzzy supply quantities $\tilde{b}_j^p$

$\tilde{b}_1^1 = (14, 16, 19, 21)$	$\tilde{b}_1^2 = (20, 23, 25, 28)$
$\tilde{b}_2^1 = (17, 20, 22, 25)$	$\tilde{b}_2^2 = (16, 18, 19, 21)$

Table 7. Fuzzy supply quantities $\tilde{e}_k$

$\tilde{e}_1 = (46, 48, 51, 53)$	$\tilde{e}_2 = (51, 53, 56, 58)$

Assume that the membership function for each $\tilde{c}_{i\,j\,k}^{r\,p}$, $\tilde{a}_i^p$, $\tilde{b}_j^p$ and $\tilde{e}_k$, $r = 1, ..., s$; $p = 1, ..., t$; $i = 1, ..., m$; $j = 1, ..., n$; $k = 1, ..., \ell$ in FMOMISTP problem take the following form:

$$\mu_{\tilde{q}_j}(q_j)=\begin{cases} 0, & q_j \le q_j^1 \\ 1-\left(\dfrac{q_j-q_j^2}{q_j^1-q_j^2}\right)^2, & q_j^1 \le q_j \le q_j^2 \\ 1, & q_j^2 \le q_j \le q_j^3 \\ 1-\left(\dfrac{q_j-q_j^3}{q_j^4-q_j^3}\right)^2, & q_j^3 \le q_j \le q_j^4 \\ 0, & q_j \ge q_j^4 \end{cases} \qquad (6)$$

For any $\alpha \in [0, 1]$, the non fuzzy problem PMOMISTP corresponding to the FMOMISTP problem takes the form:

$$\min\ (\tilde{c}_{111}^{11}x_{111}^1+\tilde{c}_{111}^{12}x_{111}^2+\tilde{c}_{211}^{11}x_{211}^1+\tilde{c}_{211}^{12}x_{211}^2+\tilde{c}_{121}^{11}x_{121}^1+\tilde{c}_{121}^{12}x_{121}^2+\tilde{c}_{221}^{11}x_{221}^1$$
$$+\tilde{c}_{221}^{12}x_{221}^2+\tilde{c}_{112}^{11}x_{112}^1+\tilde{c}_{112}^{12}x_{112}^2+\tilde{c}_{212}^{11}x_{212}^1+\tilde{c}_{212}^{12}x_{212}^2+\tilde{c}_{122}^{11}x_{122}^1$$
$$+\tilde{c}_{122}^{12}x_{122}^2+\tilde{c}_{222}^{11}x_{222}^1,\ \tilde{c}_{111}^{21}x_{111}^1+\tilde{c}_{111}^{22}x_{111}^2+\tilde{c}_{211}^{21}x_{211}^1+\tilde{c}_{211}^{22}x_{211}^2+\tilde{c}_{121}^{21}x_{121}^1$$
$$+\tilde{c}_{121}^{22}x_{121}^2+\tilde{c}_{221}^{21}x_{221}^1+\tilde{c}_{221}^{22}+x_{221}^2+\tilde{c}_{112}^{21}x_{112}^1+\tilde{c}_{112}^{22}x_{112}^2+\tilde{c}_{212}^{21}x_{212}^1$$
$$+\tilde{c}_{212}^{22}x_{212}^2+\tilde{c}_{122}^{21}x_{122}^1+\tilde{c}_{122}^{22}x_{122}^2+\tilde{c}_{222}^{21}x_{222}^1+\tilde{c}_{222}^{22}x_{222}^2)$$

subject to

$$x_{111}^1+x_{121}^1+x_{112}^1+x_{112}^1 \le \tilde{a}_1^1,$$
$$x_{211}^1+x_{221}^1+x_{212}^1+x_{222}^1 \le \tilde{a}_2^1,$$
$$x_{111}^2+x_{121}^2+x_{112}^2+x_{122}^2 \le \tilde{a}_1^2,$$
$$x_{211}^2+x_{221}^2+x_{212}^2+x_{222}^2 \le \tilde{a}_2^2,$$
$$x_{111}^1+x_{211}^1+x_{112}^1+x_{212}^1 \ge \tilde{b}_1^1,$$
$$x_{121}^1+x_{221}^1+x_{122}^1+x_{222}^1 \ge \tilde{b}_2^1,$$
$$x_{111}^2+x_{211}^2+x_{112}^2+x_{212}^2 \ge \tilde{b}_1^2,$$
$$x_{121}^2+x_{221}^2+x_{122}^2+x_{222}^2 \ge \tilde{b}_2^2,$$
$$x_{111}^1+x_{111}^2+x_{211}^1+x_{211}^2+x_{121}^1+x_{121}^2+x_{221}^1+x_{221}^2 \le \tilde{e}_1,$$
$$x_{112}^1+x_{112}^2+x_{212}^1+x_{212}^2+x_{122}^1+x_{122}^2+x_{222}^1+x_{222}^2 \le \tilde{e}_2,$$
$$c_{ijk}^{rp} \in (\tilde{c}_{ijk}^{rp})_\alpha, \qquad r=p,\ i,\ j,\ k=1,\ 2;$$
$$a_i^p \in (a_i^p)_\alpha, \qquad i=p=1,\ 2;$$

$$b_i^p \in (\tilde{b}_j^p)_\alpha, \qquad j = p = 1, 2;$$

$$e_k \in (\tilde{e}_k)_\alpha, \qquad k = 1, 2;$$

$$\sum_{i=1}^{2} (\tilde{a}_i^p)_\alpha \geq \sum_{j=1}^{2} (\tilde{b}_j^p)_\alpha, \qquad p = 1, 2;$$

$$\sum_{k=1}^{2} (\tilde{e}_k{}^p)_\alpha \geq \sum_{p=1}^{2} \sum_{j=1}^{2} (\tilde{b}_j^p)_\alpha,$$

$$x_{ijk}^{rp} \geq 0, \qquad r = p, i, j, k = 1, 2, \text{ and}$$

$$\alpha \in [0, 1].$$

For $\alpha = 0$, the α -cuts of $\tilde{c}_{ijk}^{rp}$, $\tilde{a}_i^p$, $\tilde{b}_j^p$, and $\tilde{e}_k$, $r = p, i, j, k = 1, 2$ are:

$$\left.\begin{array}{ll} 8 \leq c_{211}^{11} \leq 15 & 11 \leq c_{211}^{12} \leq 8 \\ 4 \leq c_{121}^{11} \leq 11 & 6 \leq c_{121}^{12} \leq 12 \\ 6 \leq c_{221}^{11} \leq 9 & 7 \leq c_{221}^{12} \leq 14 \\ 9 \leq c_{112}^{11} \leq 15 & 11 \leq c_{112}^{12} \leq 16 \\ 10 \leq c_{212}^{11} \leq 15 & 14 \leq c_{212}^{12} \leq 20 \\ 6 \leq c_{122}^{11} \leq 12 & 6 \leq c_{122}^{12} \leq 11 \\ 6 \leq c_{222}^{11} \leq 12 & 9 \leq c_{222}^{12} \leq 14 \end{array}\right] \qquad (7)$$

The second objective function coefficients are:

$$\left.\begin{array}{ll} 4 \leq c_{111}^{21} \leq 8 & 5 \leq c_{111}^{22} \leq 10 \\ 6 \leq c_{211}^{21} \leq 11 & 10 \leq c_{211}^{22} \leq 14 \\ 3 \leq c_{121}^{21} \leq 8 & 4 \leq c_{121}^{22} \leq 9 \\ 5 \leq c_{221}^{21} \leq 8 & 6 \leq c_{221}^{22} \leq 9 \\ 6 \leq c_{112}^{21} \leq 9 & 7 \leq c_{112}^{22} \leq 10 \\ 4 \leq c_{212}^{21} \leq 10 & 6 \leq c_{212}^{22} \leq 12 \\ 4 \leq c_{122}^{21} \leq 9 & 4 \leq c_{122}^{22} \leq 8 \\ 7 \leq c_{222}^{21} \leq 13 & 5 \leq c_{222}^{22} \leq 11 \end{array}\right] \qquad (8)$$

The supply quantities are:

$$\left.\begin{array}{ll} 22 \leq a_1^1 \leq 28 & 32 \leq a_1^2 \leq 39 \\ 30 \leq a_2^1 \leq 37 & 25 \leq a_2^2 \leq 33 \end{array}\right] \qquad (9)$$

The demand quantities are:

$$\left.\begin{array}{ll} 14 \leq b_1^1 \leq 21 & 20 \leq b_1^2 \leq 28 \\ 17 \leq b_2^1 \leq 25 & 16 \leq b_2^2 \leq 21 \end{array}\right] \qquad (10)$$

The conveyance capacities are:

$$46 \le e_1 \le 53 \qquad 51 \le e_2 \le 58 \tag{11}$$

For $w_1 = w_2 = 0.5$ and using (6), (7), (8), (9), (10) and (11) into (STP$(w))'$.

The solution is:

$$x_{111}^1 = 16 \quad x_{111}^2 = 24 \quad x_{221}^1 = 1 \quad x_{221}^2 = 9$$

$$x_{112}^2 = 1 \quad x_{122}^2 = 9 \quad x_{222}^2 = 19$$

$x_{211}^1 = x_{211}^2 = x_{121}^1 = x_{121}^2 = x_{112}^1 = x_{112}^2 = x_{121}^1 = x_{121}^2 = x_{122}^1 = x_{222}^1 = 0$, and

$z^{\min} = 591$.

The stability set of the first kind is:

$$S(w^*) = \{(w_1, w_2) \in R^2 : 5w_1 + w_2 \ge 4,\ 3w_1 + 10w_2 \ge 5,\ w_1 + w_2 = 1\}.$$

8. Conclusion

In this paper, a multi-objective multi-item solid transportation problem with fuzzy objective functions coefficients, fuzzy supplies, fuzzy demands and fuzzy conveyances has been studied. The relation between α-fuzzy efficient solution of FMOMISTP and α-parametric efficient solution of PMOMISTP has been established. A parametric analysis to characterize the set of all α-parametric efficient solution has been given. A solution procedure to determine the stability set of the first kind corresponding to one parametric efficient solution of PMOMISTP problem has been presented. A numerical example has been included in the sake of the paper for illustration. However, WINQSB computer package has been to obtain the results.

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